BRIDGING THE GAP

Between Arithmetic & Algebra

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Council for Exceptional Children

Contents

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Foreword From the Council for Exceptional Children Division for Learning Disabilities

The importance of mathematics goes beyond passing end-of-year tests. Mathematics success is related to overall school success and even college completion. A barrier for many students, especially those with disabilities and those considered at risk, is the study of algebra. The Institute of Education Sciences' *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students*, a recommendation guide for middle and high school teachers to better deliver algebra instruction (Star et al., 2015), highlighted three recommendations:

1. Use solved problems to engage students in analyzing algebraic reasoning and strategies.
2. Teach students to utilize the structure of algebraic representations.
3. Teach students to intentionally choose from alternative algebraic strategies when solving problems.

The recommendations provide excellent ideas for helping students gain conceptual understanding and procedural facility with algebraic representations and equations. However, success in algebra requires more than good instructional strategies in middle and high school; students must be adequately prepared. The purpose of *Bridging the Gap Between Arithmetic & Algebra* is to provide insights, strategies, and curricular emphases that will better prepare students to be successful in algebra.

Bridging the Gap begins with a discussion of the arithmetic to algebra gap and how to help tackle the barrier called *Algebra*. In Chapter 1, the editor of this volume, Brad Witzel, explores the foundational skills leading up to algebra along with examples of how thoughtful instructional delivery can set up future success or failure. The second chapter discusses the "big ideas" of algebra and algebra preparation. With an emphasis on number sense and rational numbers, David Allsopp, Sarah van Ingen, Orhan Simsek, and Keri Haley share sound instructional approaches and how to meet the unique challenges of students with disabilities. Highlighting research-supported instruction and interventions, Chapter 3 explains systematic and explicit instruction as it relates to mathematics. In it, Elizabeth Hughes provides examples of approaches, such as a graduated sequence of instruction, and explains how to adjust the recent fascination with discovery learning to meet the needs of students with disabilities.

Chapter 1

Students With Math Difficulties and the Arithmetic to Algebra Gap

Bradley S. Witzel

Mark hates going to school, even though he has plenty of friends and enjoys most of his classes. As a ninth grader, he has experienced something with which he was unfamiliar: failure. Although Mark had never been great at math, he usually received Bs and Cs. But now, he has his first F. Maybe it is because this new course, algebra, is so very different from his previous math courses. Or maybe it is because his teacher is just tougher or meaner. Whatever it is, Mark now hates math … and he hates school.

From international comparison studies to adult application studies, it is clear that many people in the U.S. struggle with mathematics. For example, the National Mathematics Advisory Panel (2008) found that

- 78% of adults cannot explain how to compute interest paid on a loan,
- 71% cannot calculate miles per gallon,
- 58% cannot calculate a 10% tip, and
- 45% cannot solve a word problem that requires dividing fractions.

Contrast this to the increased demand for graduates of programs in science, technology, engineering, and mathematics (STEM), which is three times greater than jobs in non-STEM careers; 80% of the fastest-growing jobs in the United States require some form of STEM background (Murray, 2013). Moreover, once employed, STEM employees enjoy an average of 26% higher wages and a decreased likelihood of joblessness than non-STEM employees (Murray, 2013). However, in the financial and engineering industries, some employers are going overseas to find qualified graduates with math and science degrees to fill positions (Kavilanz, 2012). It is not that there are no jobs for people who graduate, but that these are jobs in fields that Americans do not study.

Chapter 2

Building to Algebra: Big Ideas, Barriers, and Effective Practices

David H. Allsopp, Sarah van Ingen, Orhan Simsek, and Keri C. Haley

Traditionally, the U.S. education system has treated algebra as a distinct topic from the arithmetic taught in the elementary grades (Carraher, Schliemann, & Schwartz, 2008). This is problematic because, in reality, there is no dichotomy between arithmetic and algebra; algebra and algebraic thinking actually provide the foundations for arithmetical structures. What this means is that all teachers of elementary and middle school mathematics have a role in preparing students for success in algebra. Formal algebra includes the areas of symbols and expressions, linear equations, quadratic equations, combinatorics, finite probability, functions, and probability (National Mathematics Advisory Panel, 2008).

Now, to clear the air, we are not suggesting that elementary teachers cram algebra, as one more math topic, into an already packed math curricula. The foundation for algebra (sometimes called *early algebra)* should not be an add-on to elementary and middle school math curricula. Rather, early algebra is an approach to teaching grade-level standards in which teachers support students in seeing the algebraic structure that underlies the grade-level content. At its best, this approach opens the door for students to engage in the active *doing* of mathematics (Carraher, Schliemann, et al., 2008; Kaput, 1998, 2008). In this chapter, we support, in general, an algebraic approach to teaching mathematics from preschool through eighth grade and, in particular, equipping teachers to support students with disabilities for success in algebra.

The Essentials of Algebraic Literacy

Three early algebra "big ideas" that all teachers—preschool through eighth grade—can address in their math classrooms are number patterns, variables, and the concept of equality. These big ideas lay the foundation for becoming algebraically literate. We have already suggested that algebra underlies the structure of arithmetic, but this algebraic foundation remains hidden for students unless mathematics teachers intentionally highlight that structure and draw it out.

Chapter 3

Core Algebra Instruction

Elizabeth M. Hughes

Practice makes perfect. This commonly accepted phase misses the mark a bit; rather, *perfect* practice makes perfect. Practice—the right kind of practice—makes permanency. This distinction is important when researchers and educators consider how students acquire new skills, which is essential when teaching for mastery. In the context of this book, *mastery* means to understand a math area's concept and procedural facility. A master artist can apply his skills across different situations and contexts; similarly, a student who has mastered mathematics can use those skills and apply them to new situations and contexts. If a student learns imperfect concepts and procedures, intervention may be necessary to reteach the accurate skills. Reteaching can result in the loss of precious instructional time and places the student at risk of falling behind peers. The most efficient and effective way educators can support students' acquisition and learning of a new skill and developing their perfect practice of that skill may be through quality core instruction.

Core instruction supports students to acquire and practice new skills correctly the first time, thus reducing the chances that students will need remediation or intervention later on. Rakes, Valentine, McGatha, and Ronau (2010) emphasized that instructional strategies are important to learning algebra. Curriculum standards communicate what students need to learn, but teachers have the responsibility to determine how to deliver instruction in an efficient way that supports student learning.

Core instruction is central to primary, daily instruction; strong core instructional elements contribute to effective instruction and positive student outcomes. To identify "strong core instructional elements," researchers and educators look to practices that have a history of success for most learners, then build new successes from students' learning strengths and what students can do. *Research-supported practices* are those demonstrated to be effective through empirical research; there is documentation that they have worked in the past and are likely to work in the future when implemented with fidelity.

Core instructional strategies should have evidence supporting use for diverse learners such as those with disabilities, English language learners, and students who

CHAPTER 4

Progressions That Lead to Algebraic Success

Sarah R. Powell and Bradley S. Witzel

A high-performing young teacher of high school math was up for District Teacher of the Year. During the review process for the award, the teacher's department chair asked him to visit the "feeder" middle schools. Excited about the opportunity to share his wisdom with the middle school teachers, many of whom he blamed for his students' difficulties, he agreed to go. However, when they entered the school, he was asked to merely observe each teacher and summarize what he learned. Convinced that he would see poor instruction and inaccurate content, he entered the first room fully prepared to record all of the errors. What he observed, however, surprised him: The instruction was incredible, and the content was clearer than in his own classroom. The second and third teachers performed similarly. As he exited the school, his department chair approached him and asked what he saw. Humbled, he explained the incredible lessons that he learned from their instruction. With a grin, the chair said, "You are a good teacher. But you wouldn't appear as good if the instruction that preceded you was poor. What helped your students was not that you had some special magic, but rather that you continued the effective instruction they experienced in middle school."

As discussed in Chapter 1, the arithmetic-to-algebra gap appears to have as much to do with arithmetic and pre-algebraic understanding as it does the intricacies of algebra. As such, preparing students for algebra should start early and be a priority of elementary education. What is the current state of teaching pre-algebraic skills in elementary classrooms? How can the idea of progressions be used in elementary classrooms to set students up for success with algebra? What are some of the algebra challenges for students with learning difficulties and how can these be addressed?

CHAPTER 5

Computations in the Age of Machines

John Woodward and Mary Stroh

The increased interest in mathematics instruction for students with learning disabilities is a welcome sign, particularly given more than a decade's preoccupation with reading instruction. Research over the last 5 years has emerged in areas such as early algebraic thinking (Fuchs et al., 2008; Impecoven-Lind & Foegen, 2010), complex mathematical problem solving (Jitendra et al., 2013; Woodward et al., 2012), and even ratios and proportions (Jitendra, Star, Rodriguez, Lindell, & Someki., 2011). This work clearly indicates that special educators are moving beyond the focus on basic skills that dominated so much of the 1980s and 1990s. More important, this work comes at a critical moment when parents, educators, and politicians are considering—and intensely debating—the value of national standards and related assessments. To be sure, the instructional needs of students with disabilities should be part of this debate.

The focus of this chapter is on computations, and we pay particular attention to the way computations are taught in the intermediate and middle school grades. Put another way, we do not review the complex issues of counting, number sense, and number magnitude that are critical to kindergarten and primary-grade success (Dyson, Jordan, & Glutting, 2011; Gersten et al., 2012; Seethaler & Fuchs, 2010). Instead, we consider what might be, in the minds of many, a prosaic topic. What could be controversial about how we teach students to add whole numbers, subtract fractions, or multiply decimal numbers? After all, the traditional American algorithms are well established, and computational proficiency is a clearly articulated goal in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

In over half of this chapter, we describe two perspectives on computations: one from mathematics education and the other from special education. In the remainder of the chapter, we present examples of how the former (i.e., the mathematics education view) can be used to teach computations to students with disabilities and other students considered at risk.

CHAPTER 6

Fractions as a Stepping Stone

Bradley S. Witzel

As noted by a task group of the National Mathematics Advisory Panel (2008), "Understanding and manipulating fractions is crucial for further progress in mathematics and for tasks of everyday life" (p. 4-xv). Learning fractions is essential to elementary and middle level mathematics achievement. Beginning as early as third grade, students should be introduced to fractions notation in relation to a number line. In fourth grade, most students are taught the relationship of fractions to decimals and their equivalent forms. In fifth and sixth grade, students are to compute decimals and fractions, both positive and negative, in isolation and in context.

In a survey of 743 algebra teachers of eighth grade through high school (Loveless et al., 2008), teachers reported that student preparation was weak. Ranking student preparation on a scale from *poor* to *excellent*, most teachers scored their students' preparation as *fair*. Algebra teachers in schools with higher concentrations of minority students rated student preparation lower. These findings match more general findings from other researchers (Institute of Education Sciences [IES], 2014) who found that disadvantaged students received less effective teaching than other students, equivalent to nearly two weeks of learning math.

According to the National Survey of Algebra Teachers (Loveless, et al., 2008), when it comes to specific math skills the two lowest skills reported by middle and high school algebra teachers are word-problem approaches and rational numbers and computation involving fractions, both rated between *poor* and *fair* by teachers. One teacher commented that she was surprised by students' lack of understanding of operations and rational numbers.

In addition, Siegler and his colleagues (2012) found even more significance for fractions. Studying longitudinal data sets from the United States and the United Kingdom, they found that knowledge of fractions and division predicts students' knowledge of algebra and general mathematic in high school up to 6 years later. This result is after controlling for some of the most well-known predictors of math achievement: general intelligence, working memory, family income and background, and other math abilities. This research does not undermine the significance of such important concepts as math language, whole-number operations, and geometric advancement. However, it presents a relatively new challenge to place more emphasis on fractions development and understanding.

CHAPTER 7

Mathematical Problem-Solving

Emily C. Bouck and Mary K. Bouck

> The Williamston Middle School Student Council is planning social activities for kids to do during the summer. They find two swimming parks that will rent them the use of their facilities for a day at the following costs:
>
> Big Splash: $8.00 per person
>
> Let's Get Wet: $120 plus $5.00 per person.
>
> Which facility should the council choose if they are trying to keep cost at a minimum? Use mathematics when explaining and justifying your choice.

Problem solving is an important aspect of mathematics education; some might say that "real-world" problem solving is the ultimate goal of mathematics education (Cai & Lester, 2010). The value of problem solving is evident in the multiple standards documents written to guide mathematics education, including the National Council of Teachers of Mathematics standards documents (1980, 1989, 2000), the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and assessment programs such as the National Assessment of Educational Progress (Institute of Education Sciences, 2015; Powell, 2011). In math, *problem solving* is more than just reading a word problem, identifying the mathematics that will solve the problem, and doing the needed computation; rather, it involves thinking deeply about a challenge (Cai & Lester, 2010). Most students' ability to problem-solve in mathematics grows over time. Thus, developing mathematical problem-solving skills should start early with opportunities to engage and experience such situations often through a student's education. Educators should also create a culture of problem solving and integrate problem solving into all aspects of mathematics teaching and learning, rather than teaching problem solving as a stand-alone aspect (Cai & Lester, 2010).

CHAPTER 8

Monitoring Student Progress to Determine Instructional Effectiveness

Erica S. Lembke, Tricia K. Strickland, and Sarah R. Powell

Ongoing assessment of mathematics performance and progress is essential to teacher understanding of student mathematics achievement. Performance is typically assessed at least three times during the school year using standardized measures, and this benchmarking provides an indication of a student's proficiency in mathematics. For students whose progress is assessed more often (e.g., weekly for students with individualized education programs [IEPs] or for students at risk for mathematics difficulty), these progress checks provide an indication of how instruction is working for individual students. In the area of mathematics, similar to other academic areas, assessing performance and progress through standardized measures provides data that supports a teacher's decisions about instruction. Decisions made using data are verifiable, less subjective, and provide important documentation for communicating with parents, administrators, other teachers, and students.

Types of Progress Monitoring

When teachers think of progress monitoring, several types of measures might come to mind, including daily quizzes, teacher-made tests, and more formalized tasks available with the mathematics curriculum or through a web-based intervention or system. These progress-monitoring measures fall broadly into three categories: diagnostic measures, curriculum-based measurement (CBM) general outcome measures, and CBM skill-specific measures (see Table 8.1). The purpose of diagnostic measures is to help the teacher determine what skills a student has mastered and what skills need increased or improved instructional focus, reteaching, or intervention. In this way, diagnostic tasks serve as measures of content knowledge. Diagnostic measures might include tasks such as a page containing one type of math problem that students have recently learned, a quiz covering content from the previous day's lesson, or a common formative assessment that a sixth-grade team creates to assess weekly learning outcomes.

CHAPTER 9

Intensifying Instruction and Interventions Within Multitiered Systems of Support

Mary E. Little and Lisa A. Dieker

One of the most difficult instructional tasks is teaching to each student's level of readiness and conceptual understanding in mathematics. Recent federal policy requirements have increased the emphasis on accountability for improved achievement in mathematics for all students through effective teaching as states shift to the standards that match or closely resemble the Common Core State Standards Initiative (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) for kindergarten through eighth grade. These standards are organized by domains, clusters, and content. The five content standards include: (a) Counting and Cardinality, (b) Operations and Algebraic Thinking, (c) Number and Operations in Base Ten, (d) Measurement and Data, and (e) Geometry. The content standards are then divided into four strands that students should learn in algebra: Seeing Structure in Expressions, Arithmetic with Polynomials and Rational Expressions, Creating Equations, and Reasoning with Equations and Inequalities.

With algebra spanning the standards across grade levels, the need for teachers to respond to gaps in mathematics, specifically algebra, is critical. Currently, 26 states require a high school end-of-course exam in algebra, and the National Governor's Association (2012) noted that by the end of 2014, 45 states will require students to pass a course in algebra to graduate. To understand and master algebraic concepts, students need to develop both conceptual and procedural understanding of the foundational skills of whole numbers, operations, geometry, measurement, and fractions (National Mathematics Advisory Panel, 2008).

To teach these foundational skills and standards, knowledgeable teachers must use evidence-based instructional practices and strategies focused on student achievement (National Council of Teachers of Mathematics, 2007). In the past, students who did not respond to instruction in mathematics were often referred for evaluation for special education services as a means of responding to their needs. Today, many schools—rather than immediately evaluating a student's need for special education services—provide intensive and specialized instruction using a

CHAPTER 10

Access to Algebra for Students With Moderate and Severe Developmental Disabilities

Jenny Root, Diane M. Browder, and Bree Jimenez

Mathematical learning is pivotal to having a range of career, leisure, and daily living opportunities. Despite its importance to future functioning, if students with moderate and severe developmental disabilities receive mathematics instruction at all, it often is focused on the most basic computation skills. In a review of the research literature, Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakemanxya (2008) found 65 studies on teaching mathematics to this population of students that were published between 1975 and 2005. What these studies showed is that students with moderate and severe disabilities can learn mathematical content through interventions that use principles of applied behavior analysis, such as systematic prompting and step-by-step task analyses.

In contrast, because researchers only targeted the basics, it is not as clear from these studies precisely what students can learn. Nearly all the studies focused on numbers and operations or money skills. Sometimes the computational skills for these students focused on "chug and plug" responses, such as adding without showing any understanding of what 2 + 3 might mean. Sometimes activities were overly specific to a daily living or community routine (e.g., paying for a soda) without consideration of generalizing the mathematical concept across a wider spectrum of use. A common misconception has been that students must master readiness skills before engaging in higher order math lessons (Woodward & Montague, 2002) or master all life skills before getting any standards-based academic content instruction (Courtade, Spooner, Browder, & Jimenez, 2012). Instead, it is feasible to teach higher order, grade-aligned skills while continuing to embed early numeracy and provide opportunities for functional use of mathematics.

There are at least four reasons for moving beyond a basic focus for students with moderate and severe disabilities and specifically for teaching algebra. First, generalized use of mathematics requires that students be able to solve real-world problems. Real-world problems often involve recognizing and analyzing relationships between quantities. This recognition can be enhanced through