

How to Design and Deliver Effective Math Intervention



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T.L.L. TEMPLE FOUNDATION
EST 1962
BUILDING A THRIVING DEEP EAST TEXAS



GREATER TEXAS FOUNDATION



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Evidence-based mathematics resources for educators



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evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

~~no or negative
evidence~~

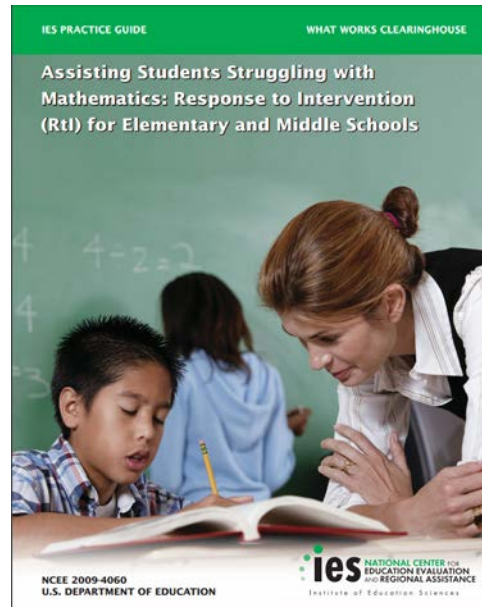
Evidence-Based Mathematics Practices


WWC Practice Guides:

- [Assisting Students Struggling with Mathematics: Response to Intervention \(Rti\) for Elementary and Middle Schools](#)

COMING SOON (2020-2021):

- [Assisting Students Struggling with Mathematics: Intervention in the Elementary and Middle School Grades](#)




 PRACTICE GUIDE

Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools



Released: April 2009









 PDF (4.0 MB)

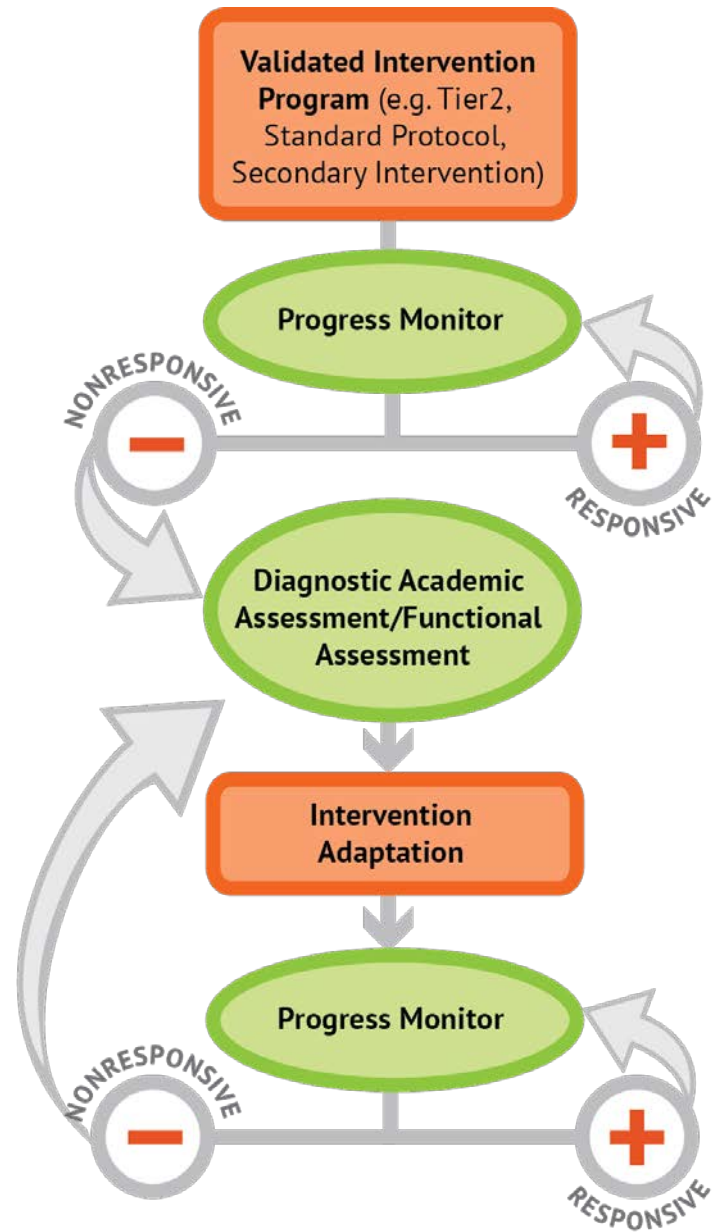
Recommendations

Details

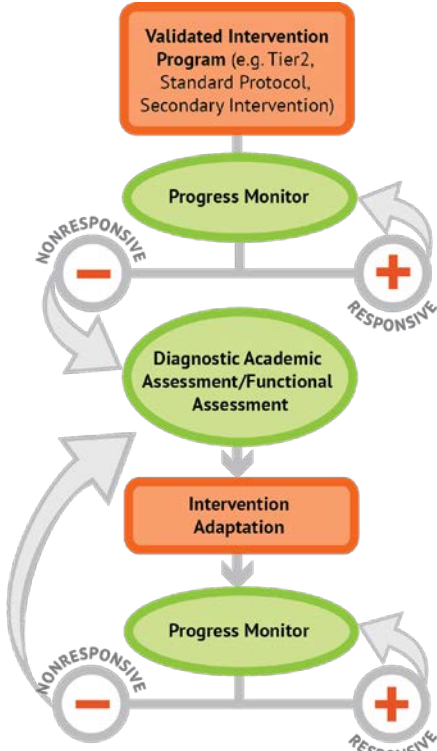
Panel

Taking early action may be key to helping students struggling with mathematics. The eight recommendations in this guide are designed to help teachers, principals, and administrators use Response to Intervention for the early detection, prevention, and support of students struggling with mathematics.

<p>1 Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.</p> <p> MODERATE EVIDENCE</p> <p>Show More</p>	<p>2 Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.</p> <p> MINIMAL EVIDENCE</p> <p>Show More</p>	<p>3 Instruction during the intervention should be explicit and systematic.</p> <p> STRONG EVIDENCE</p> <p>Show More</p>	<p>4 Interventions should include instruction on solving word problems that is based on common underlying structures.</p> <p> STRONG EVIDENCE</p> <p>Show More</p>
<p>5 Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.</p> <p> MODERATE EVIDENCE</p> <p>Show More</p>	<p>6 Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.</p> <p> MODERATE EVIDENCE</p> <p>Show More</p>	<p>7 Monitor the progress of students receiving supplemental instruction and other students who are at risk.</p> <p> MINIMAL EVIDENCE</p> <p>Show More</p>	<p>8 Include motivational strategies in tier 2 and tier 3 interventions.</p> <p> MINIMAL EVIDENCE</p> <p>Show More</p>



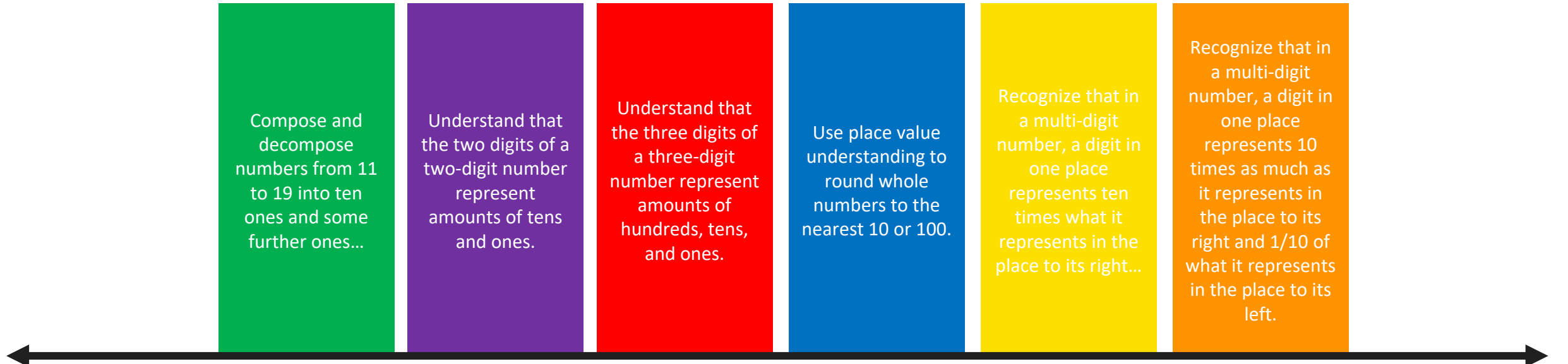
Where student IS



Where student NEEDS TO BE



Design



Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.



Where student IS

Where student NEEDS TO BE

Solve addition and subtraction word problems, and add and subtract within 10...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators
...

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Where student IS

Where student NEEDS TO BE

Explain why addition and subtraction strategies work, using place value and the properties of operations.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Use addition and subtraction within 100 to solve one- and two-step word problems...

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Apply properties of operations as strategies to multiply and divide.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Use multiplication and division within 100 to solve word problems...

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, ...

Fluently multiply multi-digit whole numbers using the standard algorithm.

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

continuum of mathematics learning

CCSS WHERE TO FOCUS GRADES K–8 MATHEMATICS

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Work with numbers 11-19 to gain foundations for place value	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
	Extend the counting sequence	Relate addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities		Use functions to model relationships between quantities
	Understand place value		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables		
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

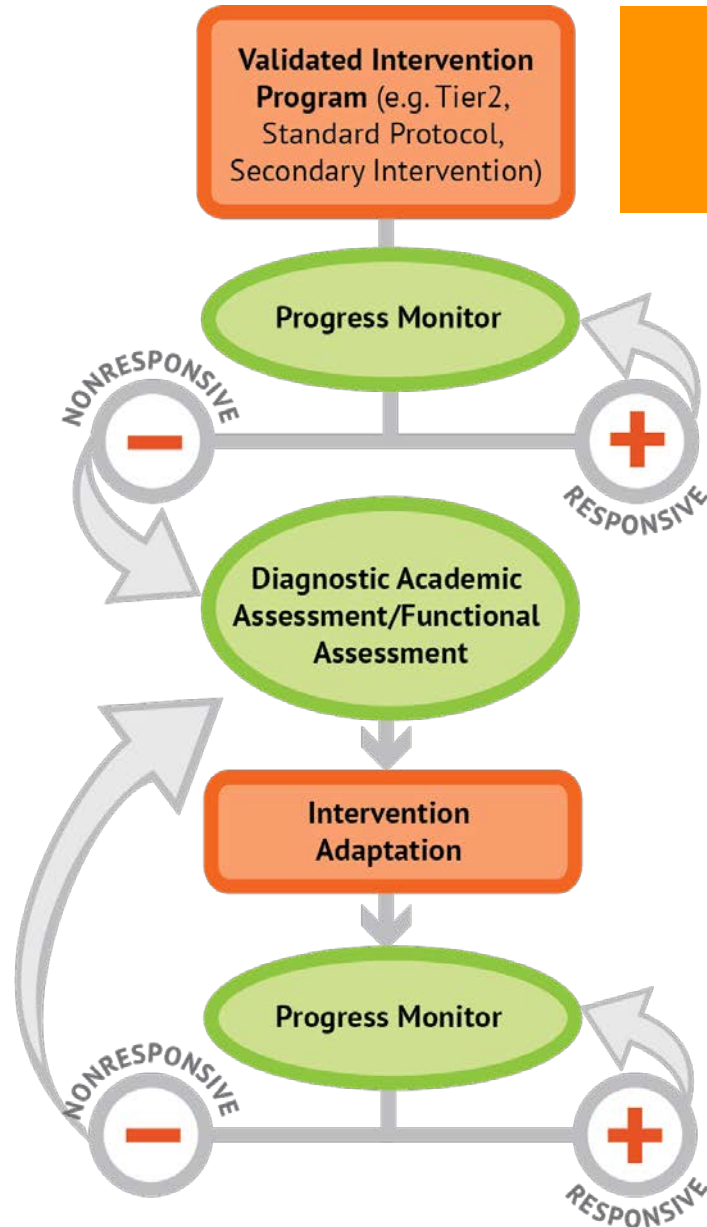
Table A.2. Grades 3–5 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*



Curriculum Focal Points and Connections	Expectations of the Content Standards
<p style="background-color: #f4a460; padding: 2px;">Grade 3 Curriculum Focal Points</p> <p><i>Number and Operations</i> and <i>Algebra</i>: Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts</p> <p>Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal “jumps” on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.</p> <p><i>Number and Operations</i>: Developing an understanding of fractions and fraction equivalence</p> <p>Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.</p> <p><i>Geometry</i>: Describing and analyzing properties of two-dimensional shapes</p> <p>Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.</p>	<p><i>Number and Operations, Grades 3–5</i></p> <ul style="list-style-type: none"> ● ● Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals ● ● ● Recognize equivalent representations for the same number and generate them by decomposing and composing numbers ● ● Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and [in Grade 6 Curriculum Focal Points] as divisions of whole numbers ● ● Use models, benchmarks, and equivalent forms to judge the size of fractions ● ● ● Recognize and generate equivalent forms of commonly used fractions, decimals, and [in Grade 7 Curriculum Focal Points] percents ● Explore numbers less than 0 by extending the number line and through familiar applications ● Describe classes of numbers according to characteristics such as the nature of their factors ● Understand various meanings of multiplication and division ● ● ● Understand the effects of multiplying and dividing whole numbers ● ● ● Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems ● ● ● Understand and use properties of operations, such as the distributivity of multiplication over addition ● ● ● Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30×50

Deliver

Instructional Platform



INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

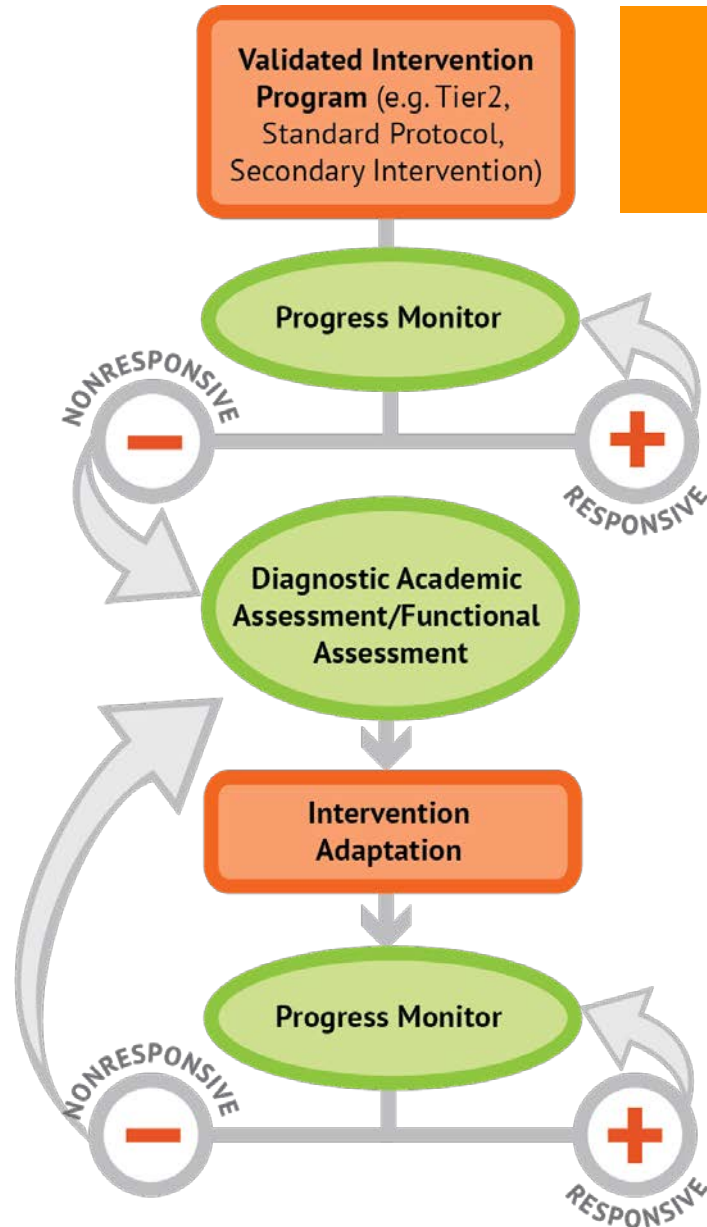
Problem solving instruction

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES



Modeling

Clear
Explanation

Planned
Examples

Practice

Guided
Practice

Independent
Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Modeling

Clear
Explanation

Planned
Examples

Practice

Guided
Practice

Independent
Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

“Let’s continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity.”

Modeling

Clear
Explanation

Planned
Examples

Practice

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Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Model steps

“To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?”

“The plus sign tells me to add. So, I’ll add 26 plus 79. I’ll use the partial sums strategy. First, I add 20 plus 70. What’s 20 plus 70?”

“20 plus 70 is 90. I write 90 right here under the equal line. Where do I write 90?”

“Then I add 6 plus 9. What’s 6 plus 9?”

“How did you add 6 plus 9?”

“6 plus 9 is 15. So, I write 15 here under the equal line.”

“Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105. What’s 26 plus 79?”

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

- Supports**
- Asking the right questions
 - Eliciting frequent responses
 - Providing immediate specific feedback
 - Maintaining a brisk pace

Goal and importance

Model steps

With examples

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

$$24 / 6$$

$$28 \div 7$$

$$35 \overline{) 5}$$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

- Supports**
- Asking the right questions
 - Eliciting frequent responses
 - Providing immediate specific feedback
 - Maintaining a brisk pace

Goal and importance

Model steps

With examples

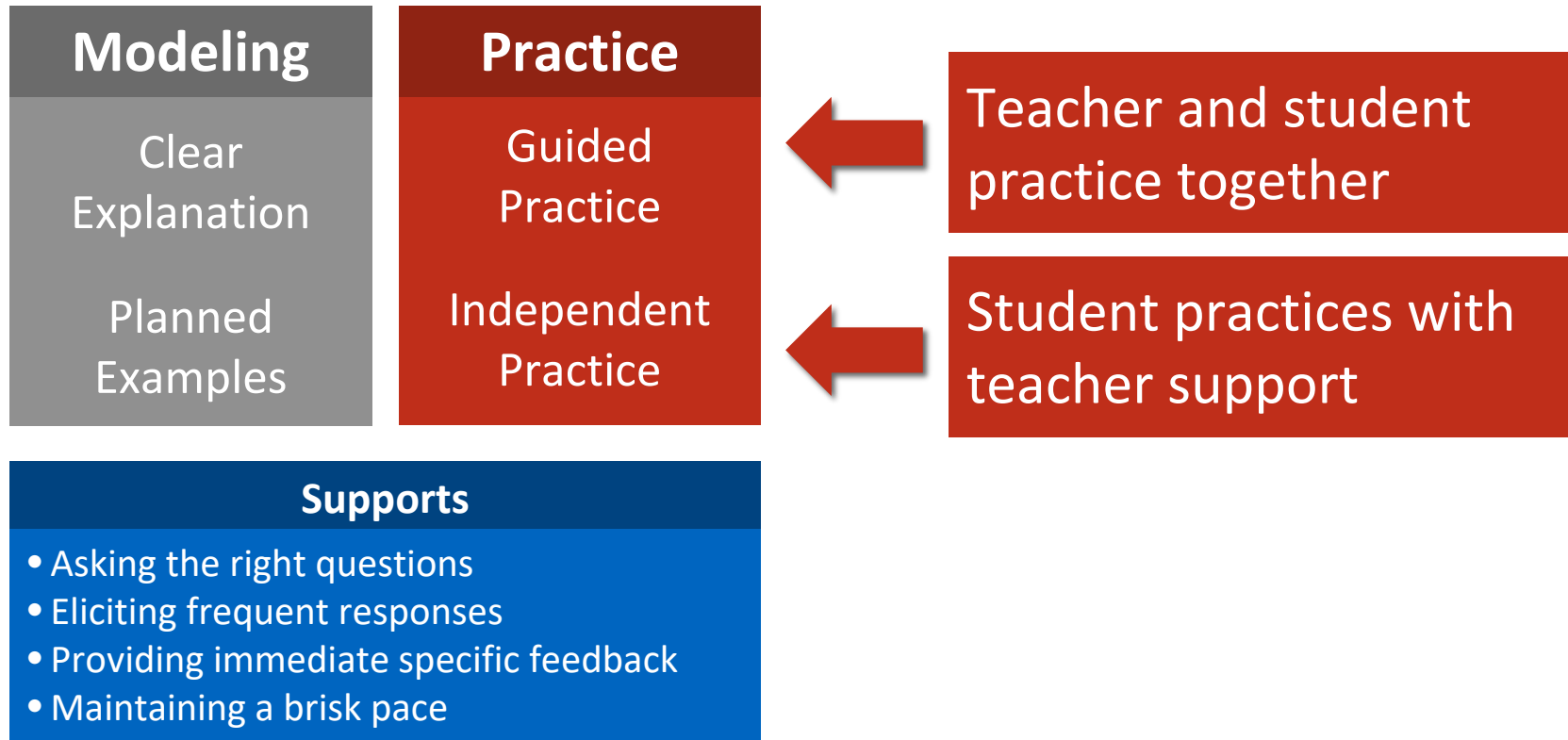
With non-examples

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

$$32 \div 8$$

$$42 \div 7$$

$$25 - 5$$



Low-level and high-level

Modeling

Clear
Explanation

Planned
Examples

Practice

Guided
Practice

Independent
Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

“What is 7 times 9?”

“Which shape has 6 sides?”

“What do you do when you see a word problem?”

“Why do you have to regroup?”

“How would you solve this problem?”

“Why do you have to use zero pairs?”

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports
<ul style="list-style-type: none"> • Asking the right questions • Eliciting frequent responses • Providing immediate specific feedback • Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

“Turn and discuss the formula for perimeter with your partner.”

“Write the multiplication problem on your whiteboard.”

“In your math journal, draw a picture to help you remember to term *parallelogram*.”

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

- Supports**
- Asking the right questions
 - Eliciting frequent responses
 - Providing immediate specific feedback
 - Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

“Good work using your word-problem attack strategy.”

“Let’s look at that again. Tell me how you added in the hundreds column.”

Modeling
Clear Explanation
Planned Examples

Practice
Guided Practice
Independent Practice

Supports
<ul style="list-style-type: none">• Asking the right questions• Eliciting frequent responses• Providing immediate specific feedback• Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Planned and organized

Modeling

Clear
Explanation

Planned
Examples

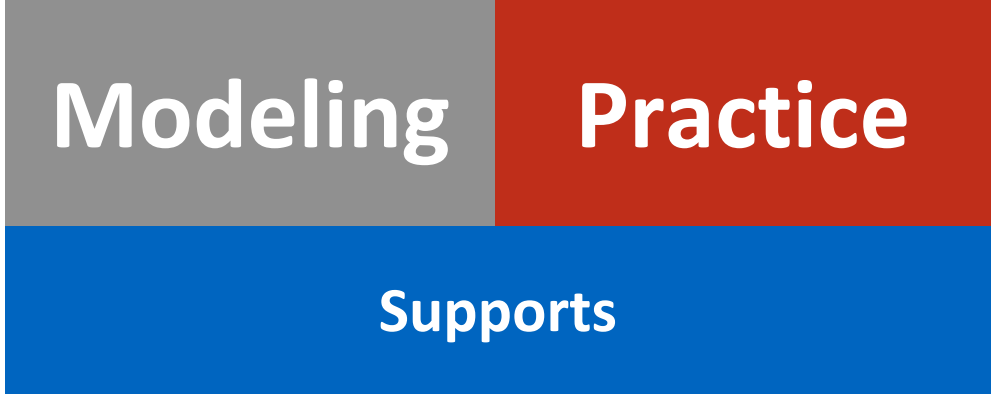
Practice

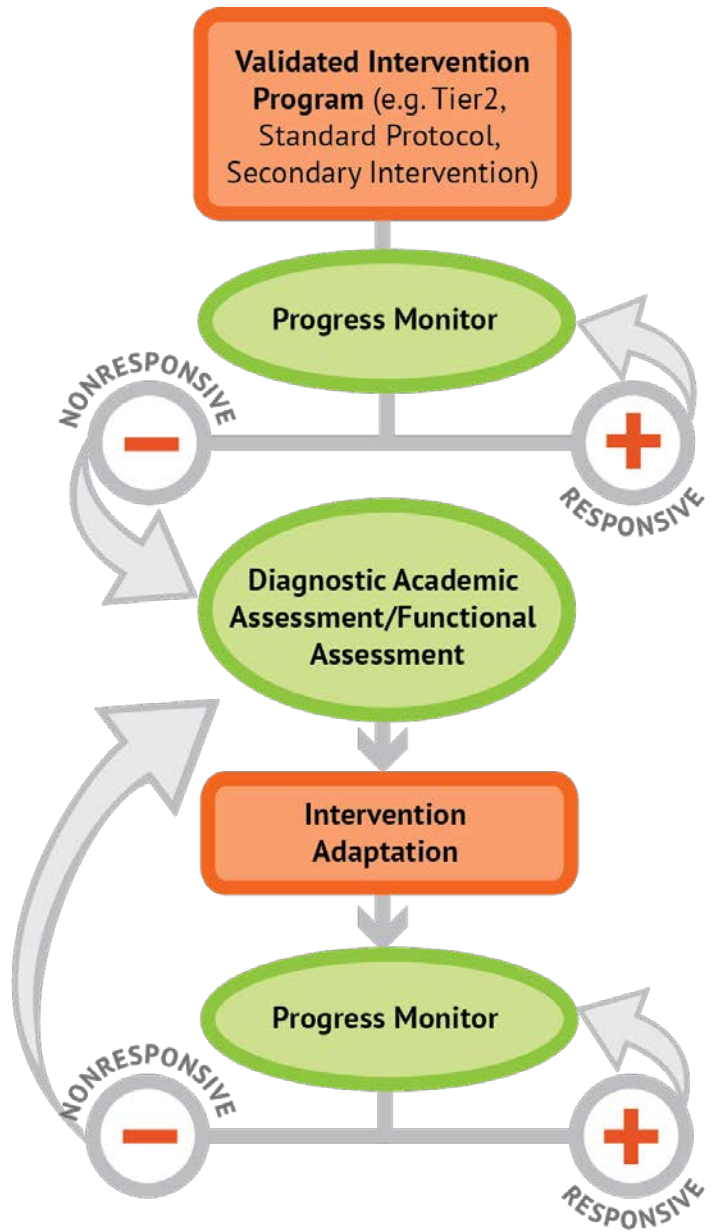
Guided
Practice

Independent
Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace





Instructional Platform

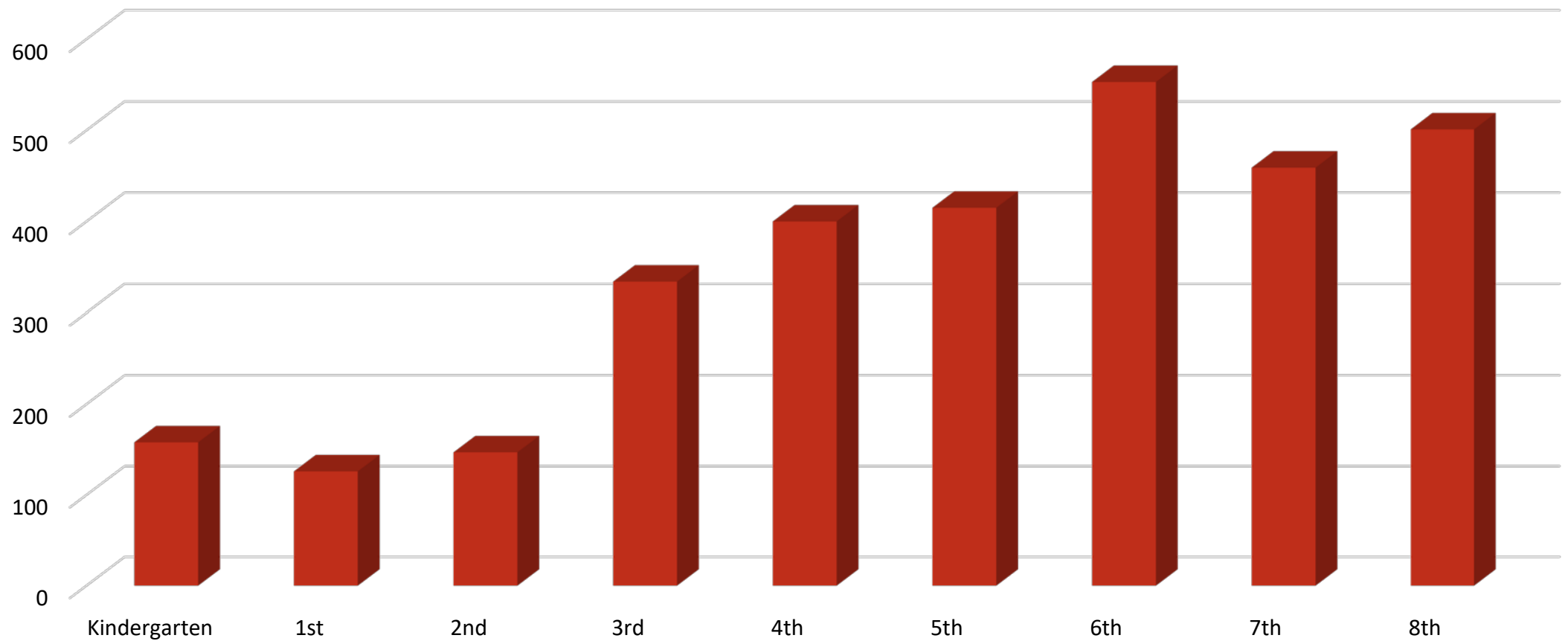
INSTRUCTIONAL DELIVERY

Explicit instruction

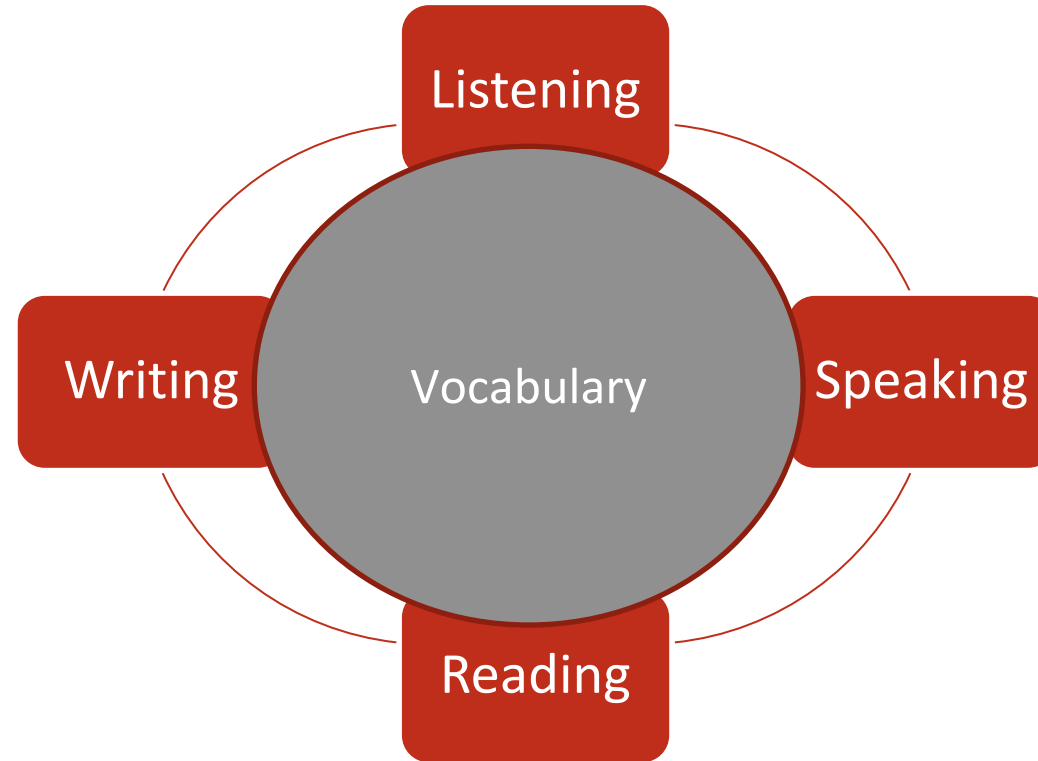
Precise language

INSTRUCTIONAL STRATEGIES

Vocabulary Across Grades



The Language of Mathematics



1. Some math terms are shared with English but have different meanings

base

right

degree

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

difference

even

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

second

square

base

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings

**divide vs.
Continental
Divide**

**variable vs.
variably cloudy**

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

factor vs.
multiple

hundreds vs.
hundredths

numerators vs.
denominator

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

mesa vs. tabla

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

four vs. forty

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

**one-fourth vs.
one quarter**

**skip count vs.
multiples**

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
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6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms

**rhombus vs.
diamond**

**vertex vs.
corner**

Use formal math language

Use terms precisely

Regroup
 Value
 Hundreds
 Ones
 Greater
 Tens
 Balance
 Place
 Digit
 Fewer

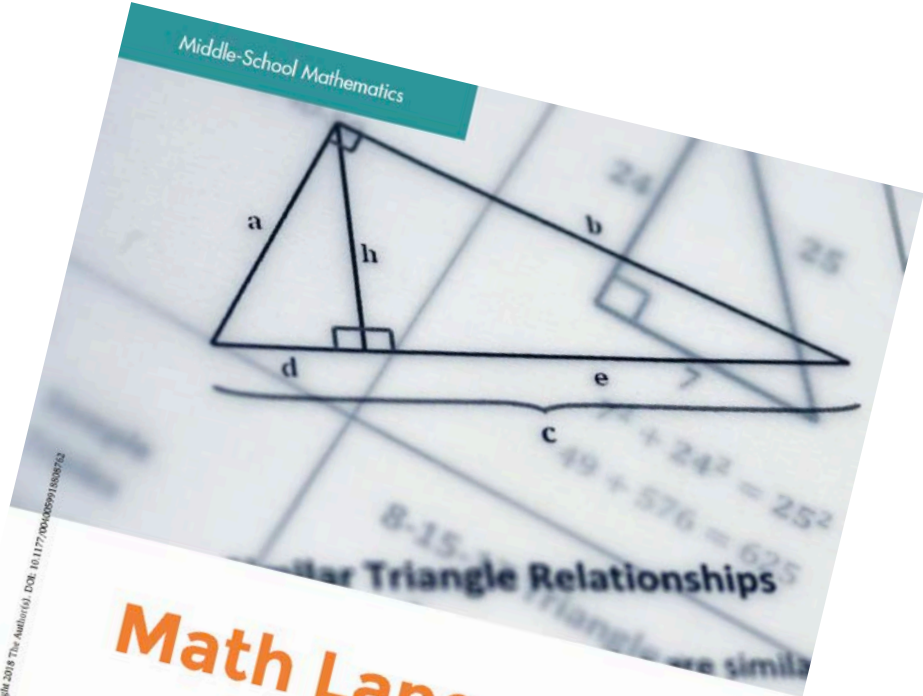


Supporting Clear and Concise Mathematics Language

Instead of That, Say This

Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens

TEACHING Exceptional Children, Vol. 49, No. 1, pp. 7-17, Copyright 2016 The Author(s), DOI: 10.1177/0048991816049001

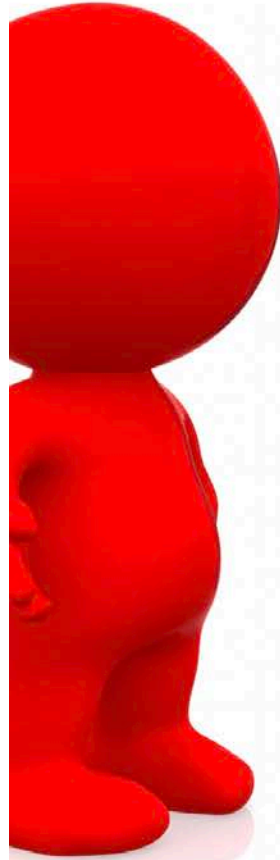


Math Language in Middle School

Be More Specific

Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes

TEACHING Exceptional Children, Vol. 51, No. 4, pp. 286-295, Copyright 2018 The Author(s), DOI: 10.1177/0048991818051004



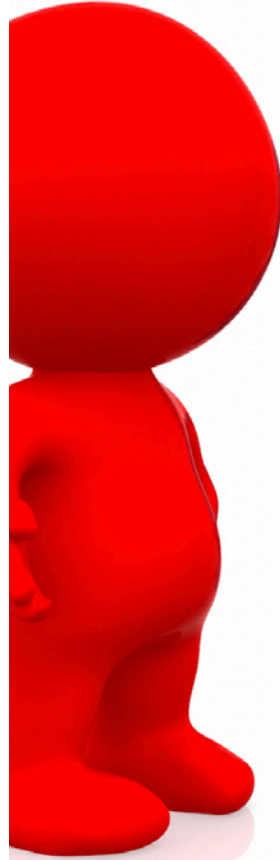
...and the last one is 10

...8, 9, 10. We'll stop counting there, but we could count more.



Why this is important...

- Suggests that 10 is the final or highest number.
- Provide opportunities to count beyond 10.
- Use language that indicates there are numbers beyond 10, but 10 is the stopping point.



What number is in the tens place?

135

What digit is in the tens place?
What is the value of the digit in the tens place?



Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



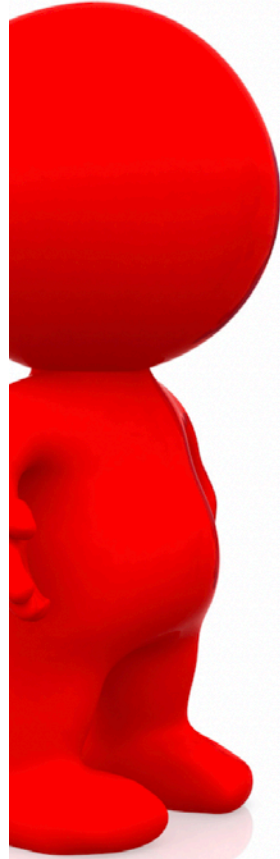
The alligator eats the
bigger number

is less than
OR
is greater than



Why this is important...

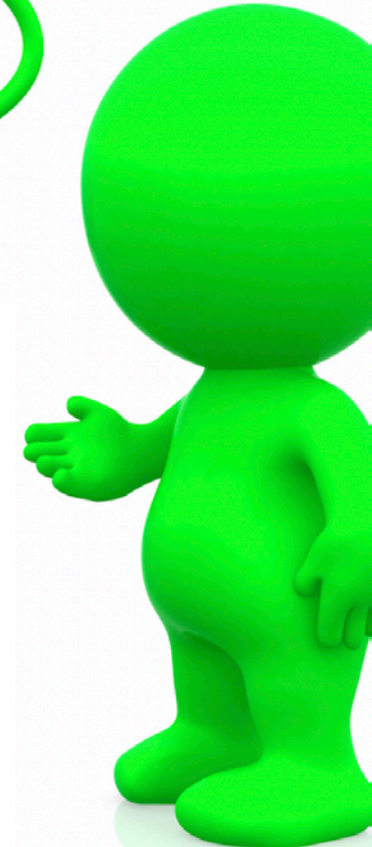
- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because $<$ and $>$ are two distinct symbols.



carry OR borrow

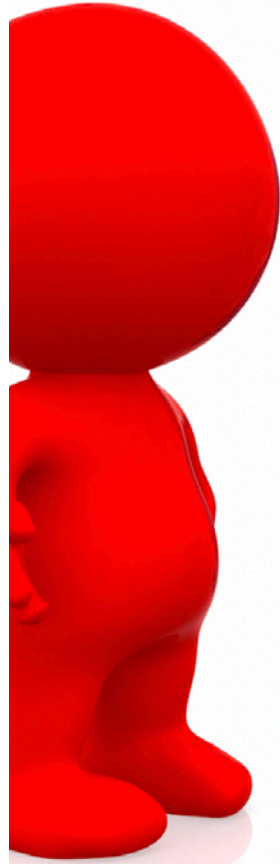
regroup OR
trade OR
exchange

$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$



Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding of regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



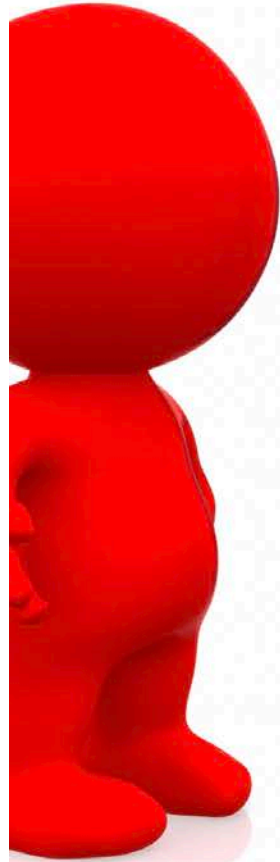
top number and
bottom number

numerator and
denominator



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



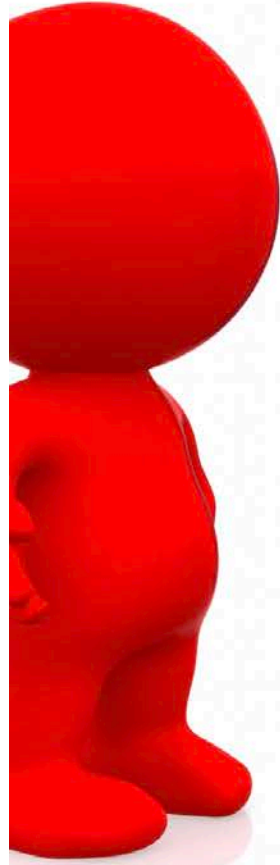
reduce the fraction

rename OR
find equivalent OR
simplify



Why this is important...

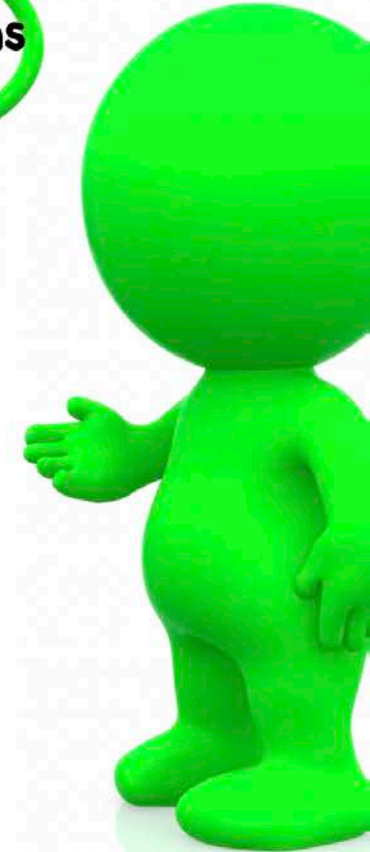
- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



Four point seven
Four point oh seven

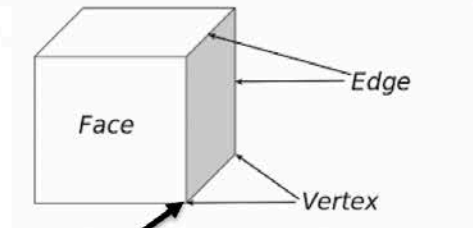
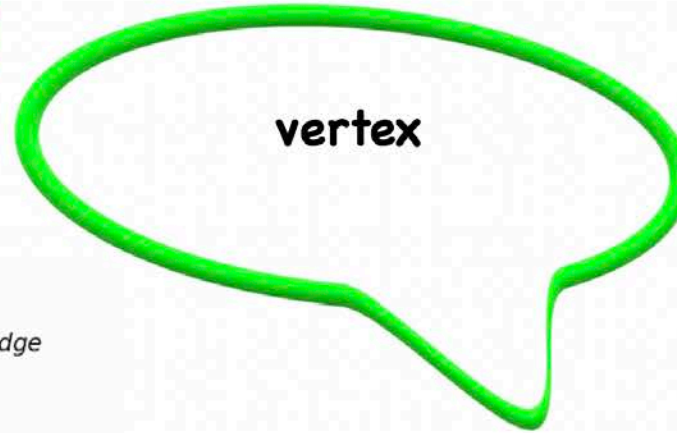
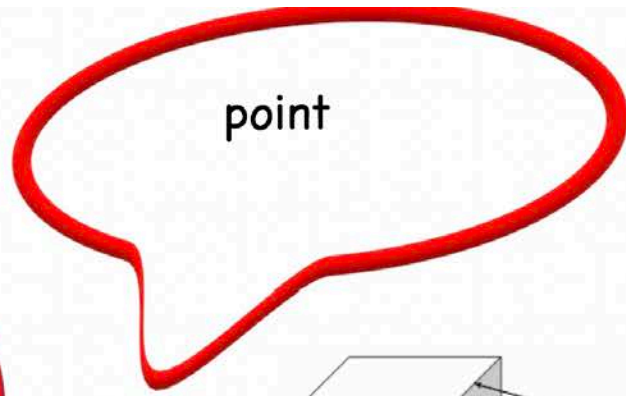
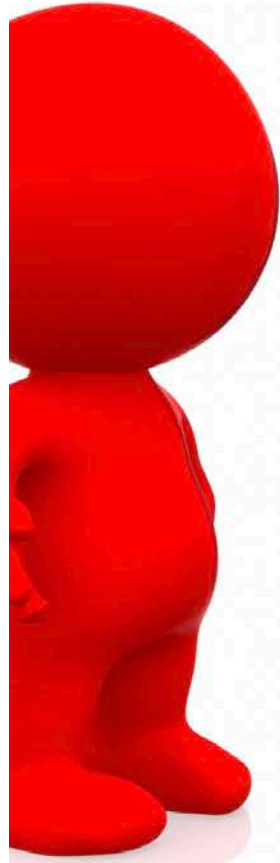
4.7
4.07

Four and seven tenths
Four and seven hundredths



Why this is important...

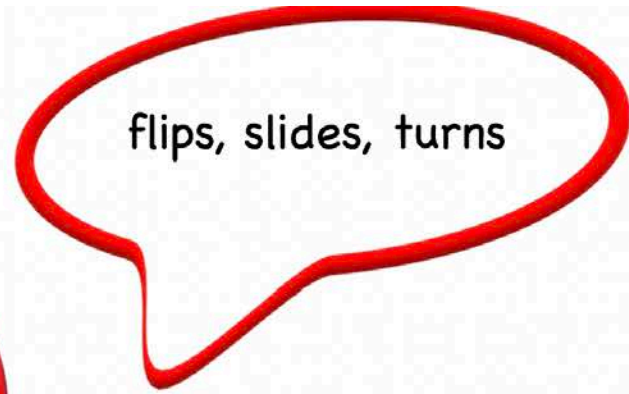
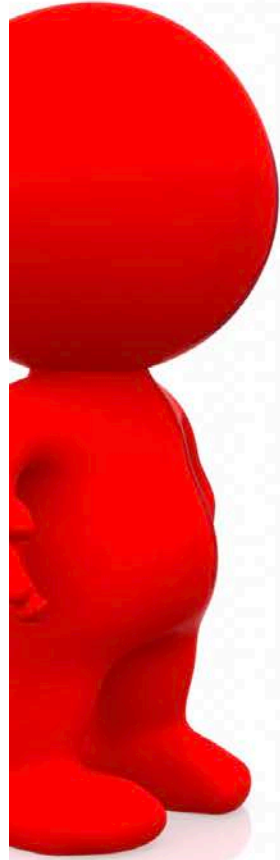
- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



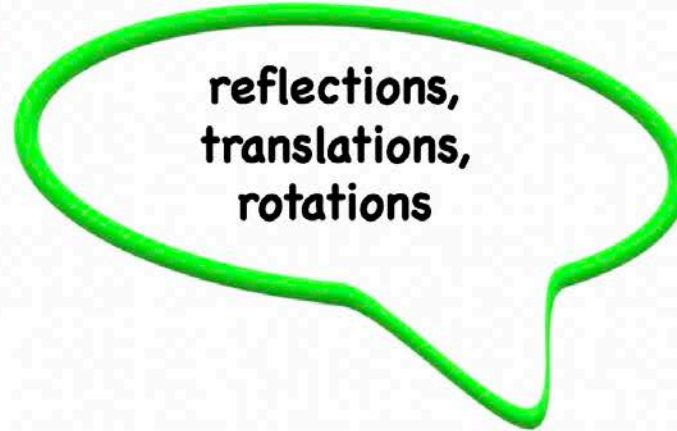
Why this is important...

- This is the endpoint where two or more line segments or rays meet.

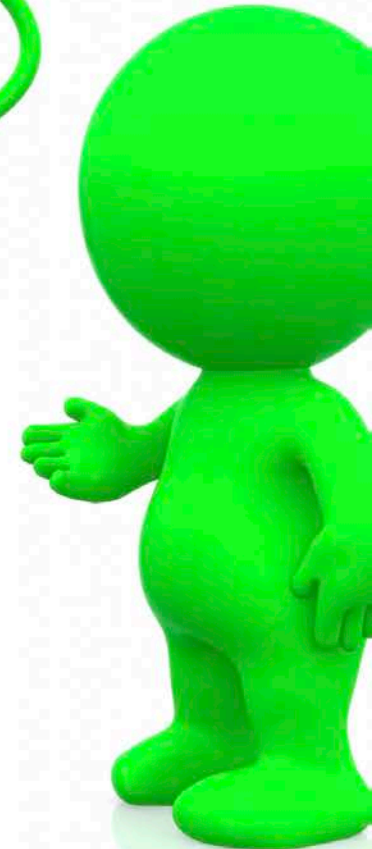




flips, slides, turns

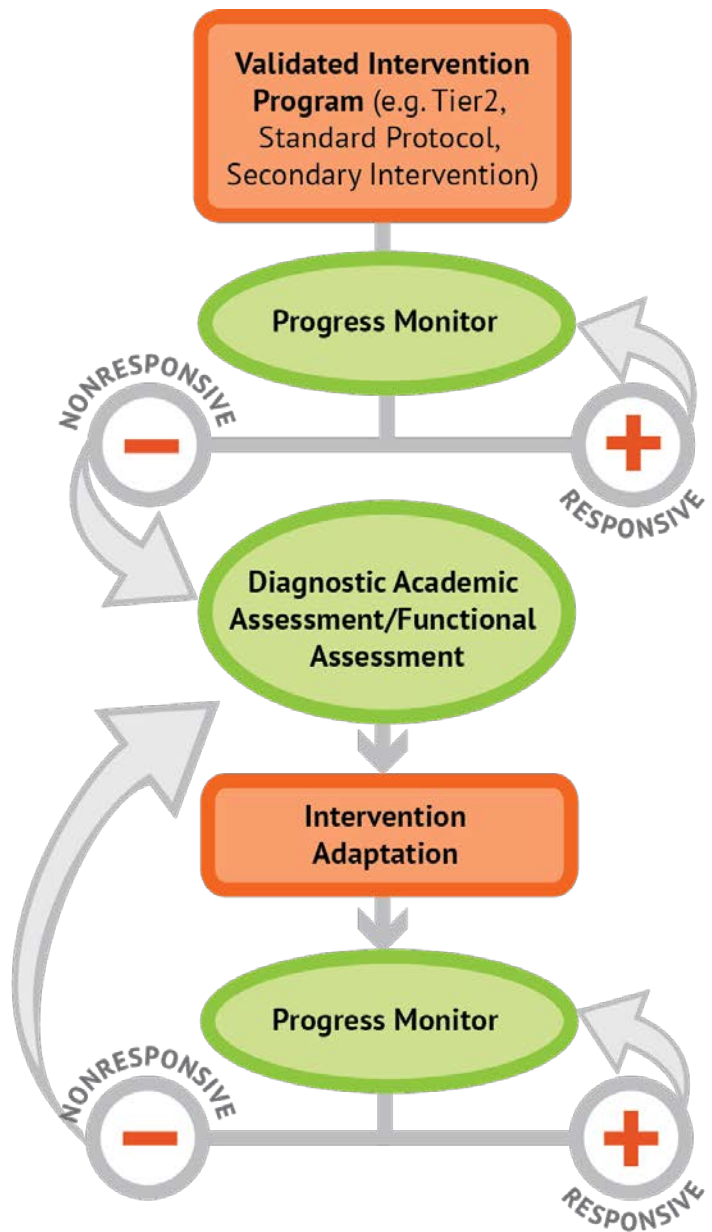


reflections,
translations,
rotations



Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Instructional Platform

INSTRUCTIONAL DELIVERY

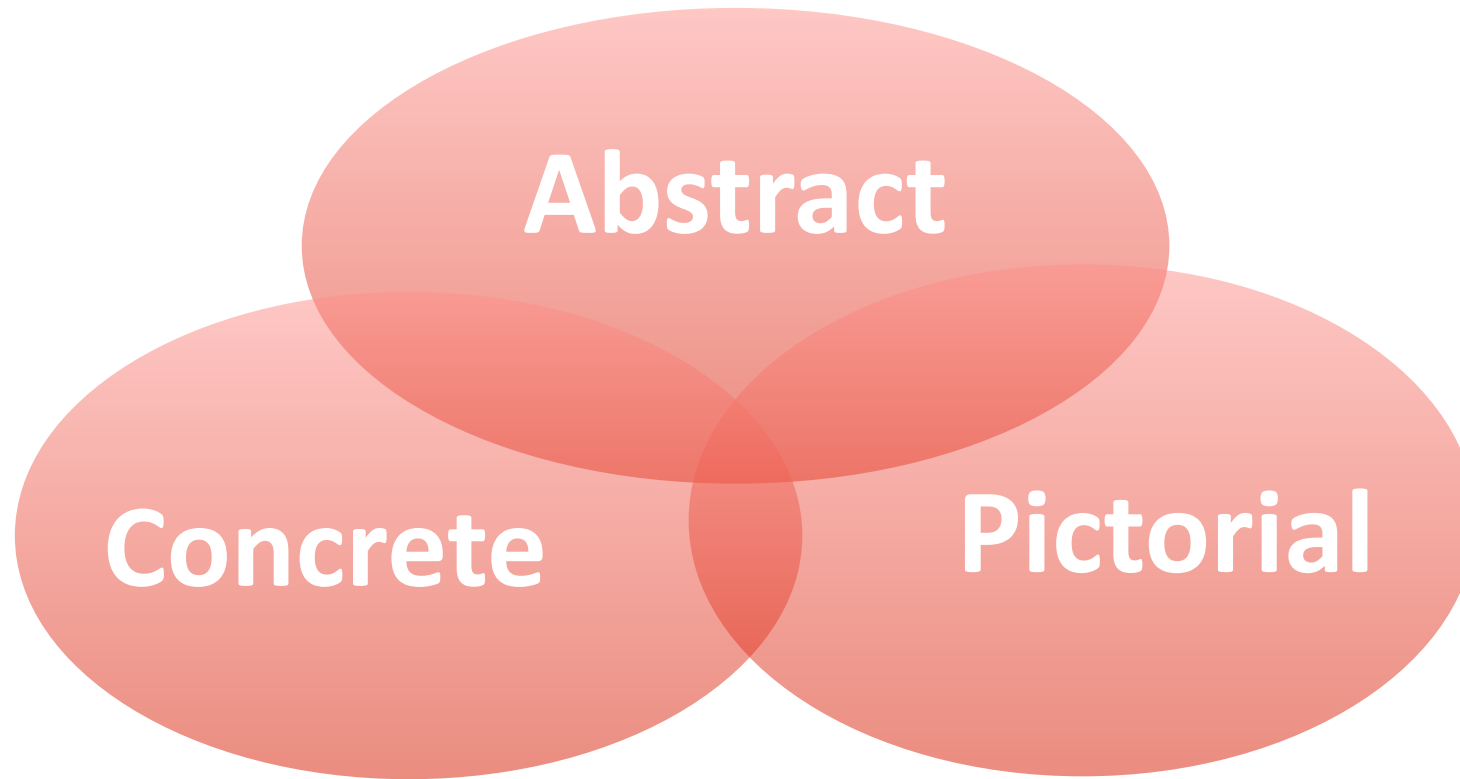
Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Multiple Representations

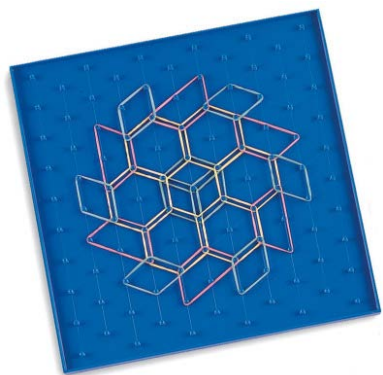


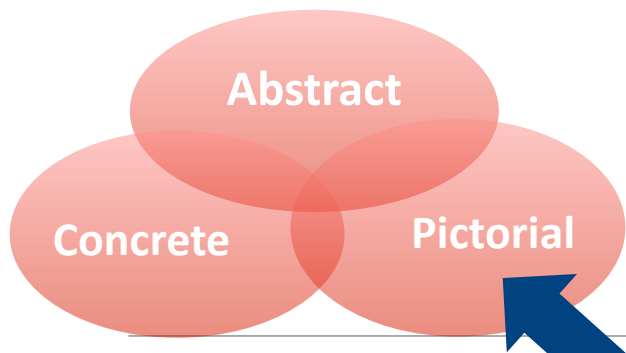
Abstract

Concrete

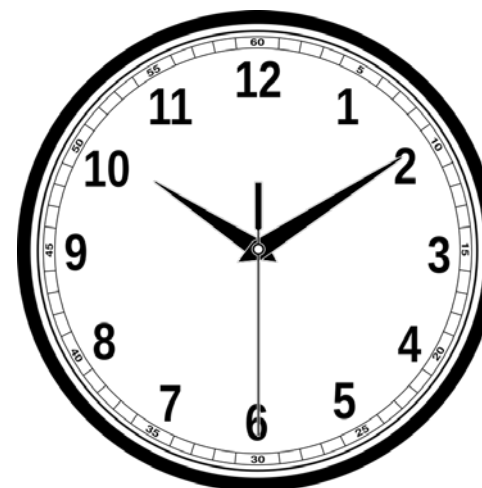
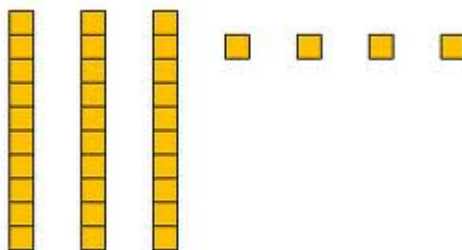
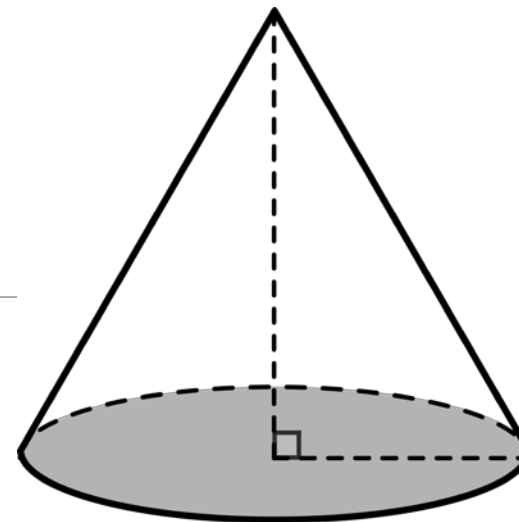
Pictorial

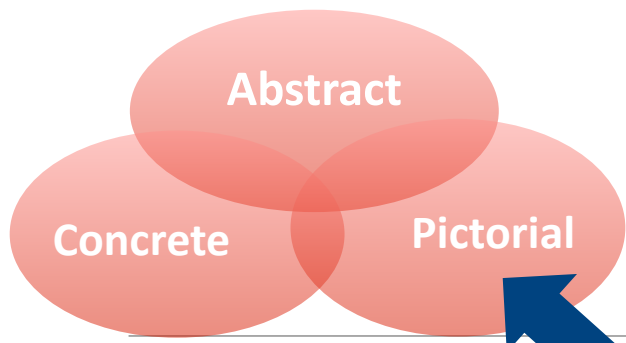
Three-dimensional objects



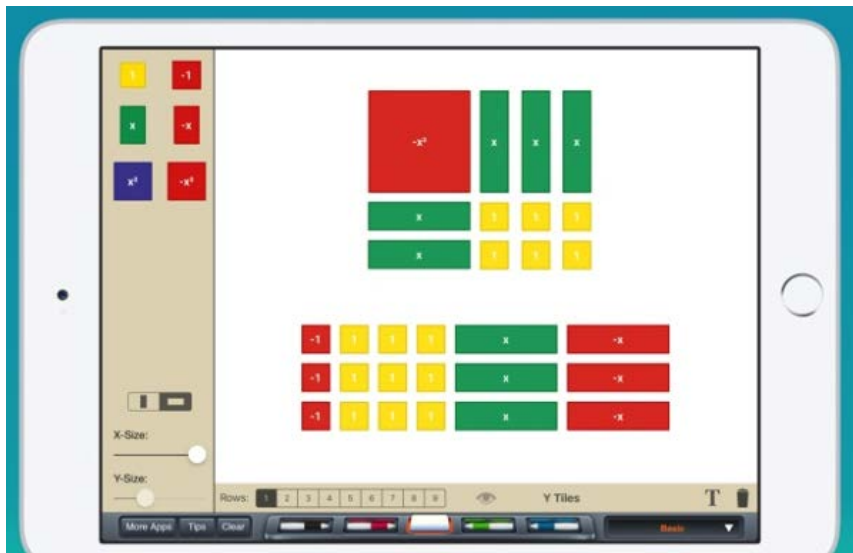
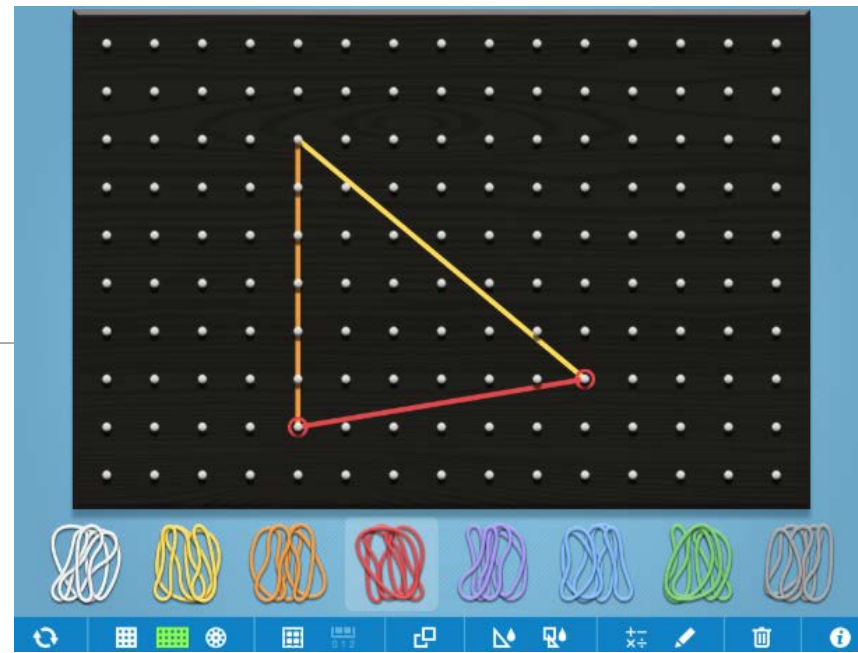


Two-dimensional images

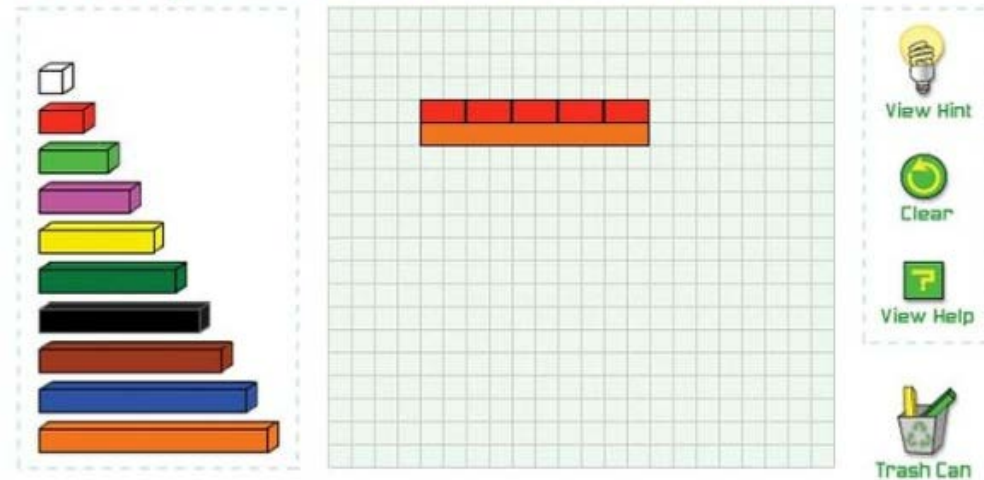


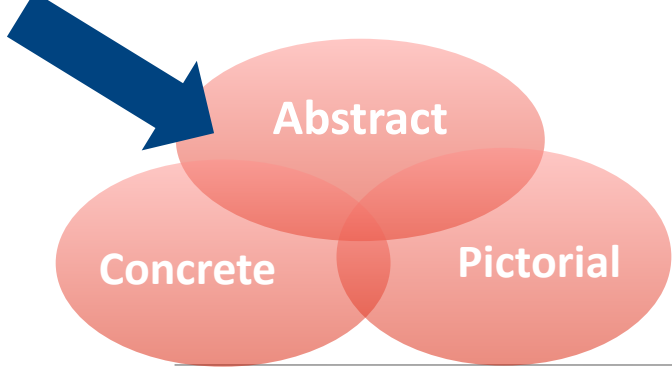


Two-dimensional images



Modeling Fractions with Cuisenaire Rods





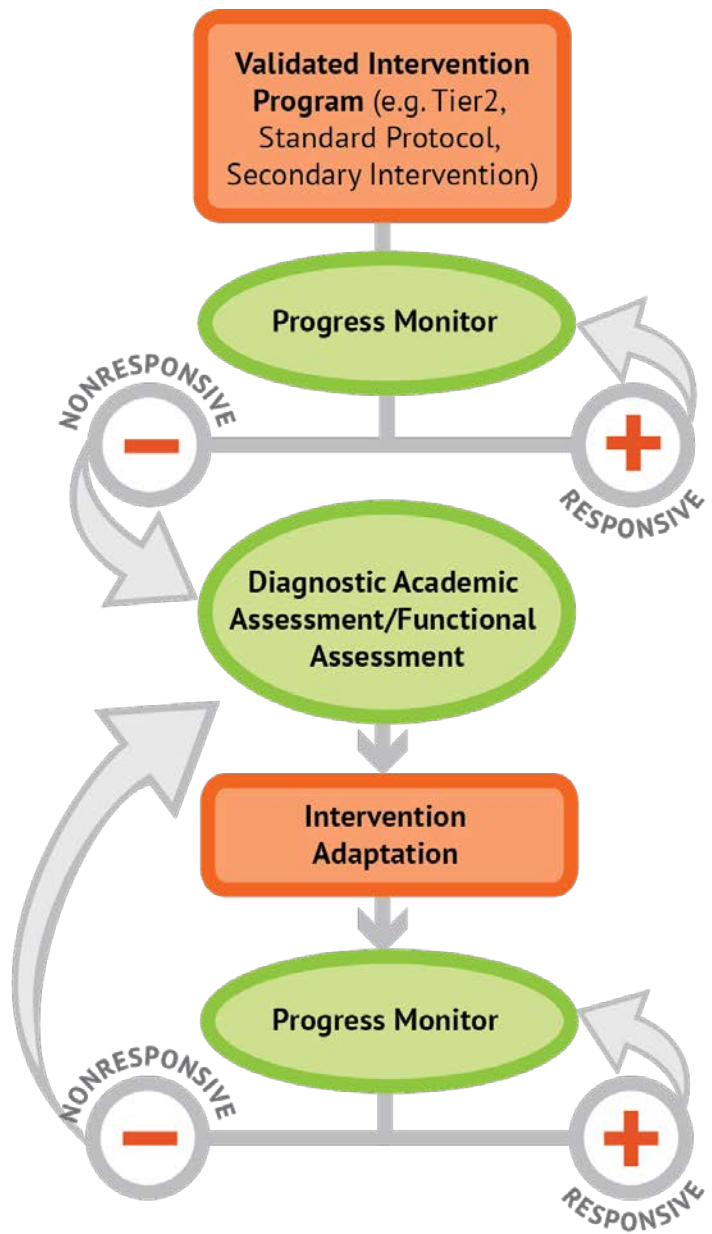
Numerals and symbols

$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$

$$x - 6 = 8$$

$$\begin{array}{r} 4,179 \\ + \underline{569} \end{array}$$



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

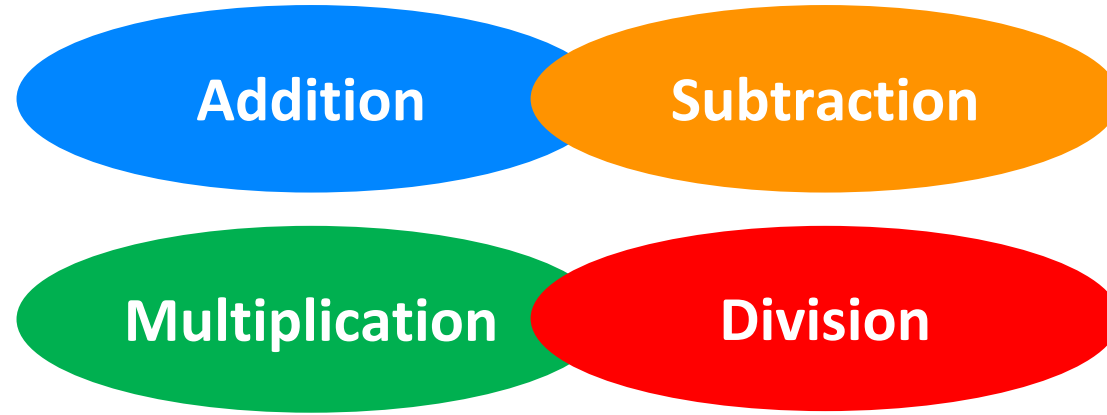
Fluency building

Addition

Subtraction

Multiplication

Division



BRIEF
(1-2 min)

DAILY
(everyday)

Cover, Copy, Compare

$$\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$$

54

$$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$$

56

$$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$$

81

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

42

$$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$$

64

$$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$$

48

$$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$$

30

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

63

$$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$$

40

$$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$$

49

File Folder

$6 + 3 =$

9

$1 + 7 =$

8

$6 + 4 =$

10

$7 + 3 =$

10

$2 + 7 =$

9

$5 + 6 =$

11

$4 + 7 =$

11

$7 + 8 =$

15

$6 + 7 =$

13

$7 + 9 =$

16

$7 + 6 =$

13

$8 + 7 =$

15

$7 + 0 =$

7

$9 + 6 =$

15

$6 + 0 =$

6

$6 + 8 =$

14

Taped Problems

$$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$$

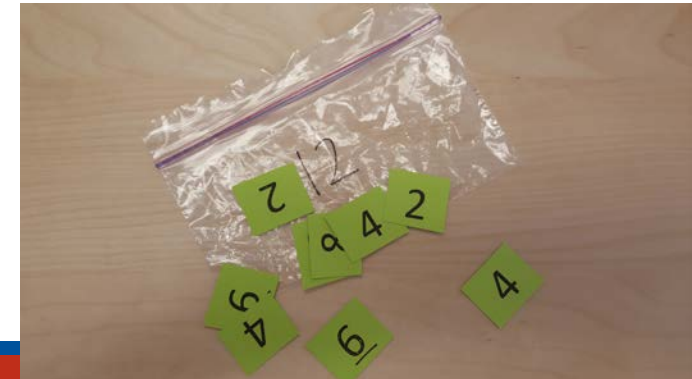
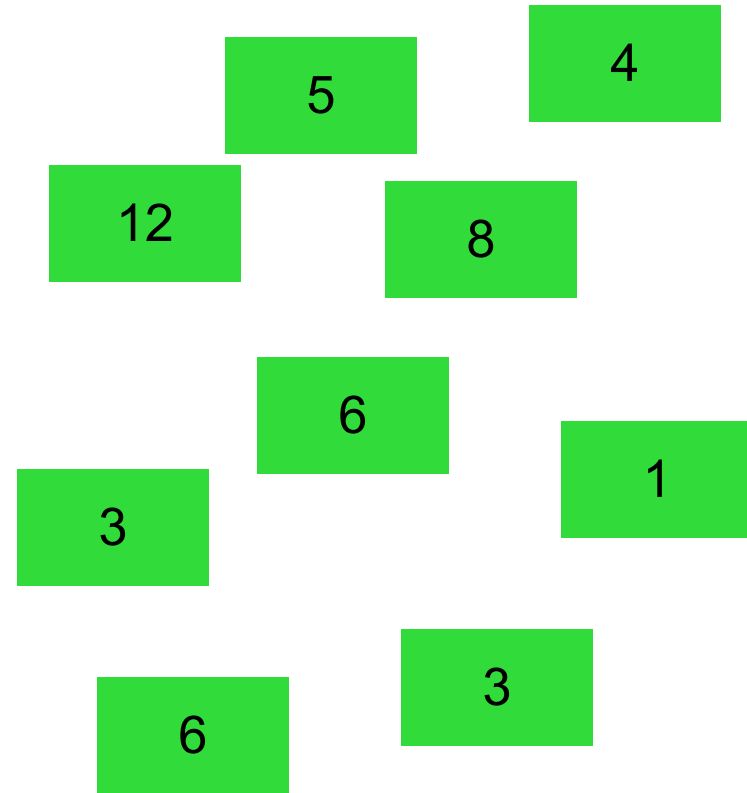
$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

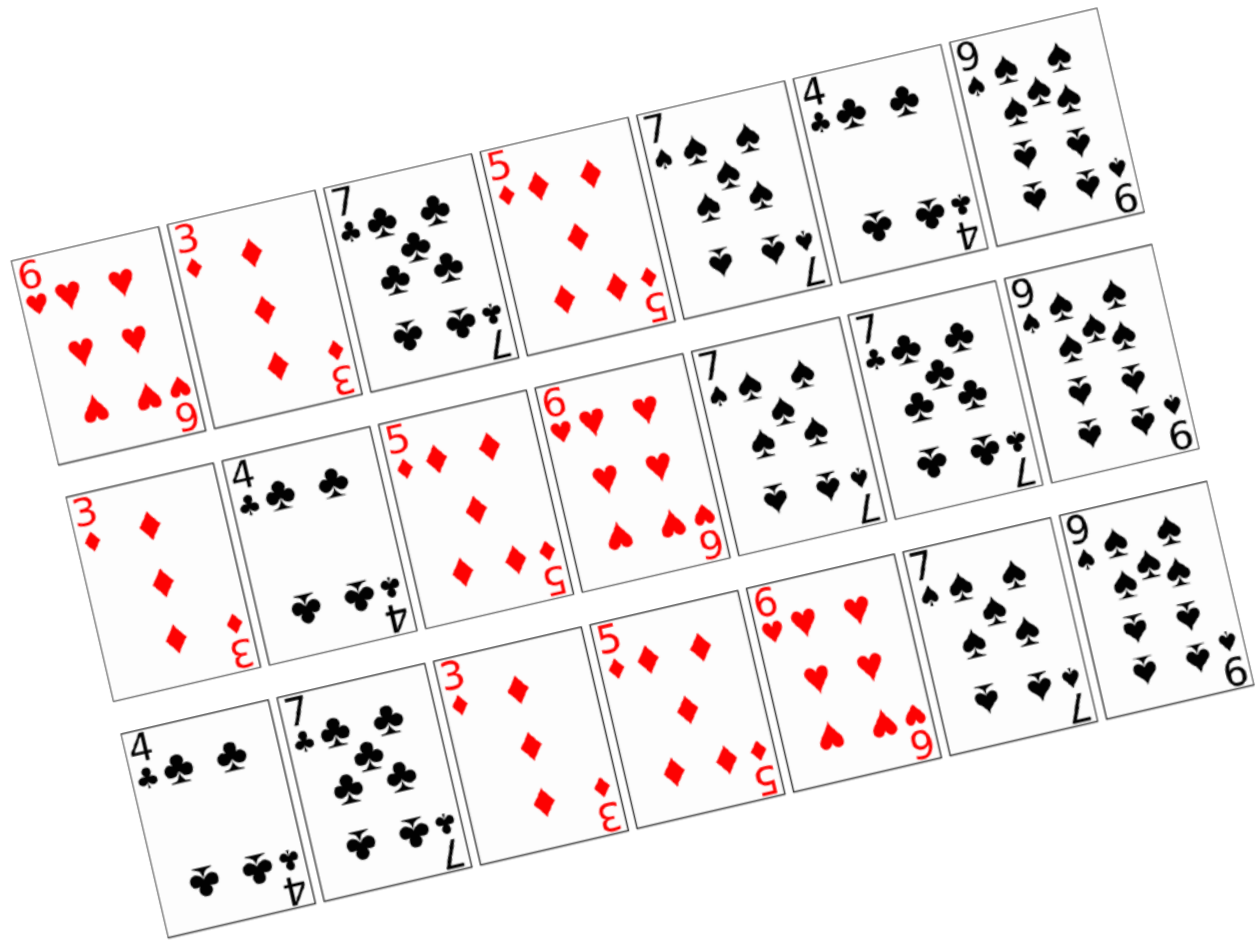
$$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$$

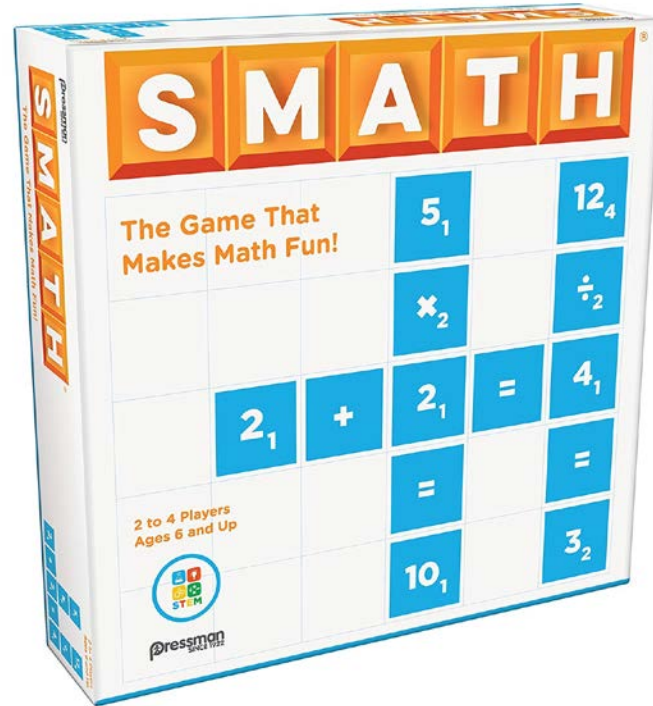
$$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$$

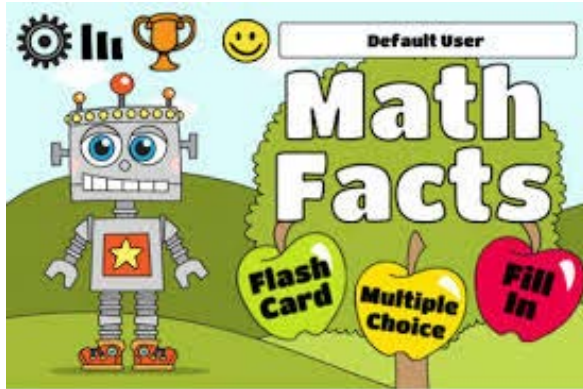
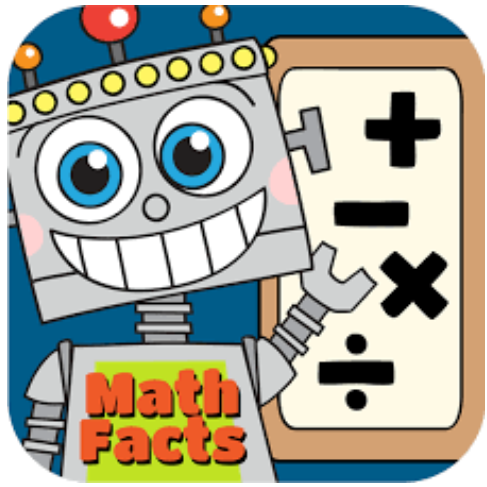
$$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$$

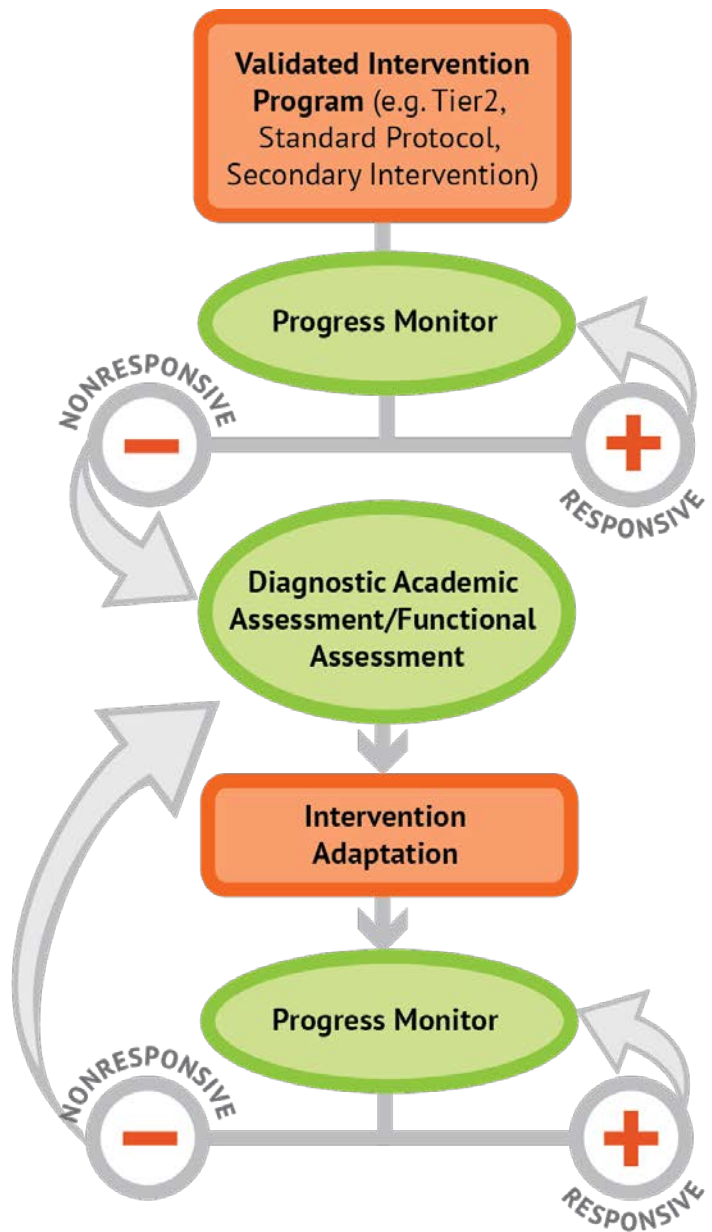
		(place sum or product from baggie here)











Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction



Don't tie key words to operations



Do have an attack strategy



Do teach word-problem schemas

Addition

total, plus, in all, join, together, sum

What it looks like:

$1 + 3 = 4$

Subtraction

minus, left, take away, fewer

What it looks like:

$4 - 1 = 3$

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> + add + all together or altogether + and + both + combined + how many in all + how much + in all + increased by + plus + sum + together + total 	<ul style="list-style-type: none"> - change - decreased by - difference - fewer or fewer than - how many are left (or have left) - how many did not have - how many (or much) more - how much longer (shorter, taller, heavier, etc.) - less or less than - lost - minus - need to - reduce - remain - subtract - take away
+	-
Multiplication Words	Division Words
<ul style="list-style-type: none"> x by (dimension) x double x each group x every x factor of x increased by x multiplied by x of x product x times x triple 	<ul style="list-style-type: none"> + as much + cut up + each group has + equal sharing + half (or other fractions) + how many in each + parts + per + percent + quotient of + ratio of + separated + share something equally
×	÷

combined

addition in all total

sum together plus

both perimeter add

more than

triple twice

factor product

multiply

each per

in all multiplied and

times double

key words

subtract decrease fewer

remain take away minus

less than how many more...

division split quotient

equal groups divide

half shared equally each

average

distribute

Addition

Sum
Total
Altogether

Ex: How much did it cost altogether?

In all
Added to
Increase / Increased by

Subtraction

minus
Less / Less than
Difference / Comparing

Ex: How much more...
How much taller...
How much further...

Decrease / Decreased by
How much is Left?

L must know how much you've used first!

Multiplication

Product
times
multiplied

Ex: He ate $\frac{2}{3}$ of the cookies

Each
Per

Ex: It cost \$1 per soda, how much for 10 sodas?

Division

Quotient
Average
Divided by

Each > Just One!

Ex: It cost \$240 for 12 apples. How much per apple?

How much will fit into a certain total...

Ex: Each shirt cost \$8, how many can you buy for \$64?

Key Words

Word Problems: Subtraction

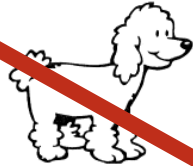
Name _____

Date _____

Subtraction

Read each problem. Write a number sentence and solve.

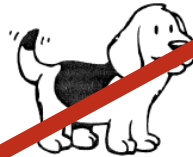
1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?



2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?



3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?



4. Mr. Green has 60 terriers. 25 of them are boys. How many terriers are girls?



5. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?



6. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?

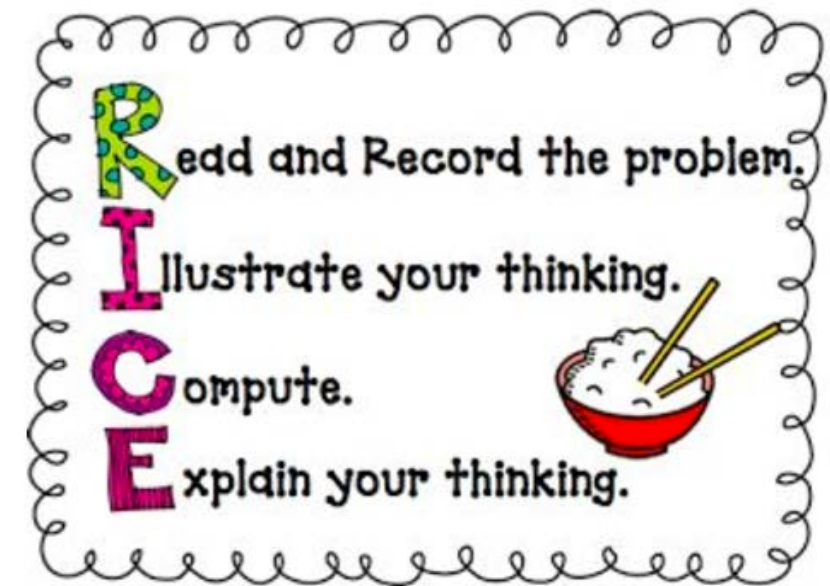


RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation
development.
Solve the
equation.

RIDE

Read the problem.
Identify the relevant information.
Determine the operation and unit
for the answer.
Enter the correct numbers and
calculate, then check the answer.



UPS✓

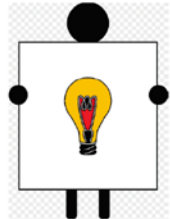


Understand

Read the problem.

Ask yourself:

- what information do I know?
- what is the question asking me to find?



Plan

Choose a strategy, a tool or an approach.



Solve

Show the math used to solve the problem.



Check

Check your math.

Ask yourself:

- Did I answer the question asked?
- IS my answer reasonable?

Total

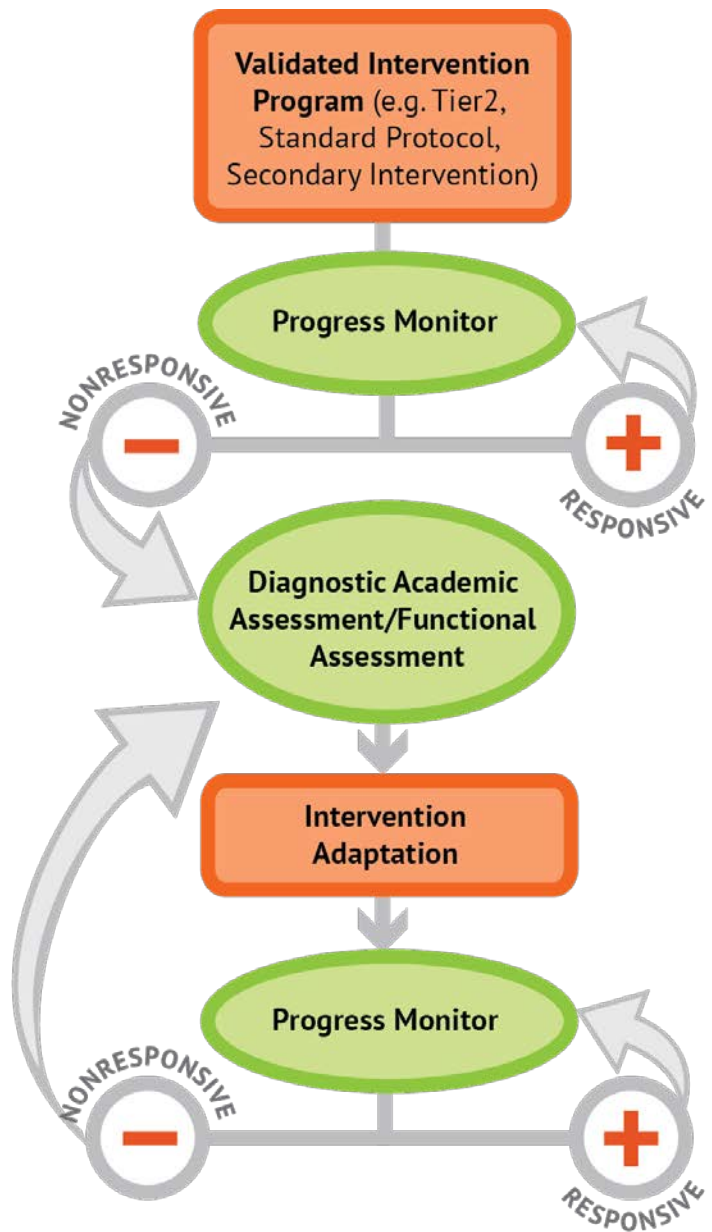
Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

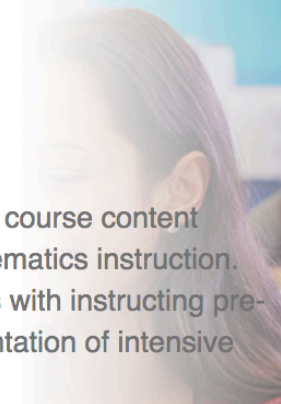
INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.



Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.

sarahpowellphd.com

Evidence-based mathematics resources for educators



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Presentations

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