## 10 Key Mathematics Practices for All Elementary Schools

with strong evidence of effectiveness from high-quality research

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## All elementary students can become proficient in mathematics if:

1. Teachers emphasize number sense. Students learn what quantities and numbers mean and how to represent them with objects and numerals. Students know the position of numbers on a number line. Students count fluently and compare amounts.


## Examples

- Students understand 5 as represented by the numeral, the number word five, five objects, and five fingers.
- Students place 14 on a number line. Students describe 14 as greater than 13 and less than 15.
- Students count objects or pictures quickly and without hesitation.
- Students count two sets of objects and compare. For example, "9 bears is more than 6 bears."

2. Teachers ensure that students have fluency with addition, subtraction, multiplication, and division math facts. Students know the 390 math facts. Until learned, students practice facts regularly. Students also develop computational fluency. Students understand procedures for multidigit addition, subtraction, multiplication, and division. Students learn fluency for computation with fractions and decimals.

## Examples

- Students answer addition and subtraction facts like $4+8$ and $10-3$ quickly and without hesitation. Fluency with addition and subtraction facts should be established by the end of second grade.
- Students answer multiplication and division facts like $3 \times 8$ and $54 \div 9$ quickly and without hesitation. Fluency with multiplication and division facts should be established by the end of fourth grade.
- Students develop efficiency with computation-adding, subtracting, multiplying, or dividing in steps. For example, $109+62$ or $6,598 \div 18$.

3. Teachers help students understand math concepts. Teachers teach the "why" and "how" of math in combination with procedures and rules.

## Example

When working to solve $246+197$, students learn why notation requires a 1 written above the tens column. Students understand that in the ones column, $6+7=13$ ones. 13 ones is regrouped to 1 ten and 3 ones. This instruction emphasizes the concept of regrouping.
4. Students are expected to use hands-on materials and visual representations to show concepts and procedures.


## Examples

- When learning about the properties of quadrilaterals, students use plastic sticks to show the differences and similarities between a square, rectangle, rhombus, kite, trapezoid, and parallelogram.
- When learning to represent the fraction $2 / 3$, students use the three models of fractions.
- Area: Rectangle divided into 3 equal parts, 2 parts shaded
- Length: $2 / 3$ placed on a number line
- Set: 3 circles, 2 shaded


5. Students are taught problem-solving strategies. Students are taught how to read problems and organize work according to the structure of the problem. Teachers do not use "key words" to teach problem solving.

## Examples

- Every time students see a mix of numbers and words (a word problem), they use a step-by-step process to solve the problem. The first step is to read the problem. Then, students make a plan, solve the problem, and check the work.
- Students learn to identify word problems by structure. For example, students identify word problems as comparison prob-
 lems (comparing two amounts), change problems (an amount changes), or combine problems (amounts combined). Students use structure to identify word problems instead of defining word problems by the operation (for example, addition or subtraction) for solution.
- Teachers avoid saying things like, "When you see the word altogether, that means you add." Altogether could be used in word problems that use addition, subtraction, multiplication, or division. Using key words is superficial and does not work across grade levels.

6. Students are expected to show work and have opportunities to explain work in oral and written forms.

## Example

Students write their steps to solve a problem about measuring the perimeter of a room.

7. Students use technology, when appropriate, to learn math concepts and practice procedures.

## Examples

- To practice properties of triangles, students use a tablet application with a geoboard to show equilateral, isosceles, and scalene triangles.
- Students practice math facts with brief, daily lessons via software.

8. Teachers provide differentiated instruction to meet the needs of all students. Teachers use explicit instruction when introducing new math content.

## Examples

- Teachers extend the math learning of students exceeding grade-level expectations and work on foundational skills with students not meeting grade-level expectations.
- Teachers model math problems step-by-step. Students participate in guided practice with the teacher and independent practice with teacher feedback. Teachers design planned and organized lessons, ask the right questions, require frequent student response, and provide affirmative and corrective feedback.

9. Teachers use precise math language. Students use correct math language when verbalizing explanations and steps for solving problems.

## Examples

- From the beginning of fraction instruction in first and second grades, teachers use the terms numerator and denomi-
 nator to describe the parts of a fraction. Teachers avoid exclusive use of the terms top number and bottom number. Throughout the elementary grades, teachers consistently describe the numerator as the parts of the known fraction and the denominator as the whole divided into equal parts.
- Instead of saying, "I put the number over here," students say, "My number was greater than 9 , so I have to exchange 10 tens for 1 hundred."

10. Teachers improve student beliefs and attitudes about math.

## Examples

- Teachers work to eliminate students saying, "I'm not good at math."
- Teachers show students how math is helpful in life. <br> \title{
10 KEY MATH PRACTICES <br> \title{
10 KEY MATH PRACTICES for All Middle and High Schools
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## with strong evidence of effectiveness from high-quality research

## All middle and high school students can become proficient in mathematics if:

1. Teachers help students to solve mathematics problems by using manipulatives and tools to bridge concrete to abstract and symbolic understandings of mathematics.
2. Students are asked to make their mathematics thinking transparent by talking about their solution process, drawing a picture, or making a graph and using mathematically correct language (for example, using the terms "numerator" and "denominator" rather than "top number" and "bottom number" for fractions, telling how many groups of a divisor are in the dividend for division rather than saying 5 "goes into" 20 four times, or using the term "zero pairs" rather than "canceling out").
3. Students are asked to read and critique one another's written responses to problems.
4. Teachers present "real-life" word problems for students to solve daily.
5. Students are expected to solve multiplication and division facts regularly as a basis for working on rational numbers and algebraic problems.
6. Students are expected to master the properties of operations (order of operations; commutative, associative, and distributive properties; multiplicative identity property; multiplicative inverse property).
7. Students are given solved problems (correctly solved and incorrectly solved using common misconceptions) to analyze and discuss how the problems were solved and where the solution strategy broke down for incorrectly solved problems.
8. Teachers differentiate mathematics instruction for diverse learners (for example, struggling learners, English language learners, gifted students, and average achievers).

9. Teachers verbalize (think aloud, describe steps for a strategy) explanations of concepts and steps for solving problems.
10. Teachers collect data regularly to determine whether their students are benefiting from instruction and use the data to make informed instructional decisions for subsequent lessons.

